

Physical Basis of the Resonant Acoustic Method for Flaw Detection

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Abstract

Resonant inspection measures the structural response of a part and evaluates it against the statistical variation from a control set of good parts to screen defects. Its volumetric approach tests the whole part, both for external and internal structural flaws or deviations, providing objective and quantitative results. This structural response is a unique and measurable signature, defined by a component's mechanical resonances. These resonances are a function of part geometry and material properties and are the basis for Resonant Acoustic Method for Flaw Detection. By measuring the resonances of a part, one determines the structural characteristics of that part in a single test. Typical flaws and defects adversely affecting the structural characteristics for powdered metal as would be introduced in the green state are geometry related and typically a result of handling. This paper introduces the physical basis of the technique.

Theoretical Background

Modal analysis is defined as the study of the dynamic characteristics of a mechanical structure or system. All structures, even structures such as metal gears or similar parts that are apparently rigid to the human eye, undergo deformation. These deformations can be described using modal analysis. Specifically, all structures have mechanical resonances, where the structure itself amplifies any energy imparted to it at certain frequencies. For example, tuning forks or bells will vibrate at very specific frequencies, their natural frequencies, for long periods of time with just a small tap. The sound that is made is directly due to these natural frequencies. In fact, any noise generated by a structure is done so by vibration, which is simply a pattern of summed sinusoidal deformations. The Resonant Acoustic Method of Non-destructive Testing (RAM-NDT) utilizes this structural dynamic behavior to evaluate the integrity and consistency of parts.

For illustrative purposes, consider the single degree-of-freedom (SDOF) mass, spring, damper system in Figure 1. It has one DOF because its state can be determined by one quantity (x), the displacement of the mass. The elements of this simplified model are the mass (m), stiffness (k)

and damping (c). The energy imparted into the system by the excitation force (f) is stored in the system as kinetic energy of the mass and potential energy of the spring and is dissipated by the damping. The mathematical representation of the SDOF system, which is called its equations of motion, is given in Equation (1) below.

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t) \quad (1)$$

The solution to the equation of motion produces an eigenvalue problem which yields the undamped natural frequency as

$$\omega_n = \sqrt{\frac{k}{m}} \quad (2)$$

Equation (2) reveals the natural frequencies, or resonances, of a system that are determined by its mass (i.e., volume and density) and stiffness (i.e., Young's modulus and cross-sectional geometry). While Equation (2) holds only for an SDOF system, the underlying relationship of mass and stiffness can be generalized for more complex systems. That is, an increase in stiffness will increase the natural frequency and an increase in mass will decrease the natural frequency. For example, consider the strings on a guitar. The larger diameter strings (more mass) produce lower tones than the smaller strings (less mass). Also, a string has a higher pitch when tightened (increased stiffness) than when loosened (decreased stiffness). It is these fundamental properties of the resonances of a system that RAM NDT utilizes to evaluate the integrity and consistency of parts.

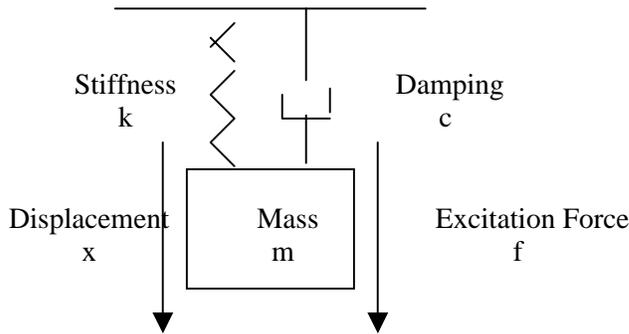


Figure 1. Single Degree of Freedom (SDOF) discrete parameter model

The natural frequencies are global properties of a given structure and the presence of structural defects causes shifts in these resonances. For example, a crack will change the stiffness in the region near the crack and a variation in density or the presence of porosity will change the mass. A crack defect typically reduces the stiffness in the material, thus decreasing the natural frequency. Similarly, porosity in a cast part reduces mass, thus increasing the natural frequency. These shifts are measurable if the defect is structurally significant with respect to the either the size or location of the flaw within a specific resonance mode shape. With some defects, a shift in resonant frequency can also be noticed audibly, such as a cracked bell that does not ring true.

Relationship to green-part induced flaws

The RAM-NDT method is particularly well suited detect several flaws which are typically introduced to the manufacturing process while the part in the green state.

These flaws are:

- geometric and symmetry deviations
- cracks induced as a result of press settings
- uneven density distribution
- material contamination

Each of these flaws is readily detectable using the RAM-NDT method by virtue of the fact that each of these flaws results in a change in resonant signature of the sintered part.

Modal Parameters and Material Properties

We are concerned with the relationship between modal parameters and intrinsic material properties. There is a direct correlation between stiffness and the modulus of elasticity, E. Hence, an account for what determines E is given below. First, however, is some general elasticity theory, which will put E in a context, but also helps in understanding the section about material damping.

Elasticity [1]

When a load is applied to a material, deformation occurs because of a slight change in the atomic spacing. The load is defined in terms of *stress* σ , which is typically in units of pounds per square inch (psi) or megapascals (Mpa). The deformation is defined in terms of *strain* ϵ , which is typically in units of inches (or centimeters) of deformation per inches (or centimeters) of the initial length or in percent.

The amount and type of strain is dependent on the atomic bond strength of the material, the stress, and the temperature. Up to a certain stress limit for each material the strain is reversible; that is, when the stress is removed, the atomic spacing returns to its original state and the strain disappears. This is referred to as *elastic deformation*, and the stress and strain are related by a simple proportionality constant. For tensile stress

$$\sigma = E\epsilon \quad (3)$$

and the proportionality constant E is called the modulus of elasticity or *Young's modulus*. For shear loading

$$\tau = G\gamma \quad (4)$$

where τ is the shear stress, γ the shear strain, and G the proportionality constant, referred to as the *shear modulus* or the *modulus of rigidity*.

At ambient and intermediate temperatures for short-term loading, most ceramics behave elastically with no plastic deformation up to fracture. This is known as *brittle fracture* and is one

of the most critical characteristics of a ceramic that must be considered in design for structural applications.

Metals also behave elastically up to a certain stress, but rather than fracture in a brittle manner like ceramics, most metals deform in a ductile manner as the stress is further increased. This is referred to as *plastic deformation* or *plastic strain* and is not reversible. Some metals, like aluminum for instance, have a smooth transition from elastic strain to plastic strain. Others, low-carbon steel for instance, have a discontinuity at the onset of plastic strain. This is called the yield point.

Not all ceramics behave in a brittle fashion and not all metals behave in a ductile fashion. Most ceramic materials undergo plastic deformation at high temperature. Pure metals have the greatest degree of ductile behavior. Addition of alloying elements reduces ductility to the point where some metals are brittle at room temperature. Cast iron is a good example. Intermetallic compounds also have little or no ductility at room temperature.

Modulus of Elasticity [1]

The modulus of elasticity E is the proportionality constant between elastic stress and elastic strain and can be thought of simply as the amount of stress σ required to produce unit elastic strain ϵ .

$$E = \sigma/\epsilon \quad (5)$$

The magnitude of the elastic modulus is determined by the strength of the atomic bonds in the material. The stronger the atomic bonding, the greater the stress required to increase the interatomic spacing, and thus the greater the value of the modulus of elasticity. Ceramics with weak ionic bonding have low E values. Ceramics with strong covalent bonding have high E values.

Metals show a similar trend. Metallic bonding is the predominant bond mechanism for metals. It is also referred to as *electronic bonding*, from the fact that the valence electrons (electrons from unfilled shells) are freely shared by all the atoms in the structure. At any given time, each atom has enough electrons grouped around it to satisfy its need for a full outer shell. It is the mutual attraction of all the nuclei in the structure to this same cloud of shared electrons that results in the metallic bond.

Because the valence electrons in a metal distribute themselves uniformly and because all the atoms in a pure metal are of the same size, close-packed structures result. Such close-packed structures contain many slip planes along which movement can occur during mechanical loading, producing the ductility that we are so accustomed to for metals. Pure metals typically have very high ductility and can undergo 40 to 60% elongation prior to rupturing. Highly alloyed metals such as the superalloys also have close-packed structures, but the different-size alloying atoms disrupt movement along slip planes and decrease the ductility. Superalloys typically have 5 to 20% elongation.

Metallic bonding occurs for elements to the left and in the interior of the periodic table. Metals with weaker bonding, such as aluminum or magnesium, have low elastic modulus. The elastic modulus of aluminum is 69 GPa (150,075 ksi). Metals with intermediate bond strength have intermediate E values. For example, most iron alloys and Ni-based or Co-based superalloys have

and elastic modulus of around 200 GPa (30,000 ksi). Metals like tungsten and molybdenum have strong atomic bonds and high values of elastic modulus.

Bond strength, and thus E, varies in different crystallographic directions. Elastic anisotropy occurs in metals as well as ceramics and in cubic as well as less-symmetrical crystal structures. A single crystal of iron, which has the body-centered cubic structure, has an elastic modulus of 283 GPa (41,035 ksi) in the [111] direction and 124 GPa (17,980 ksi) in the [100] direction. The [111] direction involves atoms that are most closely packed and have the highest net bond strength. This results in a higher E value.

Most metals and ceramics are polycrystalline and are made up of many crystals (grains) in random orientation. If the orientation is truly random, the overall elastic modulus is an average of the elastic moduli for various crystallographic directions. The elastic modulus values most commonly reported for metals and ceramics are average values for polycrystalline bodies. Even though these polycrystalline metals and ceramics have an apparent single elastic modulus, the reader must be aware that the individual crystals within the microstructure are anisotropic and that internal stresses may be present that can affect the application of the material.

The elastic modulus is affected by temperature. E decreases slightly as the temperature increases. This results from the increase in the interatomic spacing due to thermal expansion. As the interatomic spacing increases, less force is necessary for further separation.

Many materials encountered by an engineer are made up of more than one composition or phase and have an elastic modulus intermediate between the moduli of the two constituent phases. In cases where the elastic modulus value is not available, it can be estimated using the law of mixtures:

$$E = E_a V_a + E_b V_b \quad (6)$$

where E_a and E_b are the elastic moduli of the constituents, V_a and V_b the volume fractions, and E the estimated elastic modulus of the mixture. This is a simplified relationship and is suitable only for rough estimates.

Porosity also affects the elastic modulus, always resulting in a decrease. MacKenzie has derived a relationship for estimating the elastic modulus of porous materials:

$$E = E_0(1 - 1.9P + 0.9P^2) \quad (7)$$

where E_0 is the elastic modulus of a nonporous material and P the volume fraction of pores. This relationship is valid for materials containing up to 50% porosity and having a Poisson's ratio of 0.3. Most PM components contain 5-10% porosity. Hence, the strength of the final component will be lowered.

Elastic modulus measurement

There are many techniques available for to measure the elastic modulus. One of them involves direct measurement of strain as a function of stress, plotting the data graphically and measuring the slope of the elastic portion of the curve. This technique can be conducted accurately at room temperature using strain gauges, but is limited at temperatures above which strain gauges can be reliably attached. [1]

The other techniques of measuring dynamic Young's Modulus, E use, in some form or another, the basic wave equation for the propagation of a longitudinal elastic wave in an elastic medium [2]:

$$E = \rho v^2 \quad (8)$$

where ρ is the mass density of the medium and v is the wave speed. Thus the methods that are concerned with measurements of transit time t (and hence velocity) of ultrasonic pulses over a known distance L in an elastic medium apply Eq 8 directly ($v = L/t$), assuming that ρ is known or can be measured also. In the case of the methods that utilize measurements of resonant frequency of standing or decaying elastic waves in an elastic medium a modified form of Eq 8 is applied:

$$E = \rho v^2 = \rho (f\lambda)^2 \quad (9)$$

where f is the resonant frequency and λ is the wavelength. The specific geometrical details of the specimen usually determine λ . Again in the frequency methods ρ must be known or measured.

There is a frequency method relevant to our technique based on impulse excitation of vibration, in which the resonant frequency of the material is measured and E is calculated from the equation [1]:

$$E = CMf^2 \quad (10)$$

where C is a constant depending on the specimen size and shape and on Poisson's ratio, M is the mass of the specimen, and f is the frequency of the fundamental transverse (flexural) mode of vibration. This technique can be used accurately over the complete temperature range and for the various crystallographic directions of single crystals as well as for the average elastic modulus of polycrystalline material.

Material Damping [3]

The damping capacity of a material is a measure of the energy that is dissipated in the material during mechanical vibration. Most of the frequently used metals and alloys exhibit a low damping capacity.

Definitions

Material damping is a property related to time-dependent elasticity. Elasticity theory of crystalline metallic materials dictates that the relationship between an applied load and the resultant deformation obeys Hooke's Law, i.e. the resultant strain is proportional to the applied stress. Hooke's law, however, does not account for the time effect, that is, the applied load and the resultant deformation are assumed to be perfectly in phase, a condition valid only when the loading rate is so low that the deformation process may be considered to be instantaneous and static. In practice, metals and alloys respond to an applied load, not only by an instantaneous elastic strain that is time-independent, but also by a strain that lags behind the applied load, which is time-dependent (relaxation or anelasticity). Therefore the overall strain, ϵ , consists of two parts: one part, ϵ_e , is the elastic strain, and the other, ϵ_a , the anelastic strain, i.e.

$$\varepsilon = \varepsilon_e + \varepsilon_a \quad (11)$$

$$\varepsilon_a = \varepsilon_i[1 - \exp(-t/\tau)] \quad (\text{for loading}) \quad (12)$$

$$\varepsilon_a = \varepsilon_i \exp(-t/\tau) \quad (\text{for unloading}) \quad (13)$$

where ε_i is the initial strain upon loading, t is the time and τ is the characteristic relaxation constant which characterizes the anelasticity of a material. Because of the lag induced by the relaxation, the stress σ and strain ε may be expressed as:

$$\sigma = \sigma_0 \exp(i\omega t) \quad (14)$$

$$\varepsilon = \varepsilon_0 \exp[i(\omega t - \phi)] \quad (15)$$

where σ_0 and ε_0 are the stress and strain amplitudes, respectively; ω is the angular vibration frequency; and ϕ is the loss angle by which the strain lags behind the stress. By combining these two equations, the resultant complex modulus, \underline{E} , is defined as:

$$\underline{E} = \sigma/\varepsilon = \sigma_0/\varepsilon_0(\cos\phi + i \sin\phi) = E' + iE'' \quad (16)$$

where

$$E' = \sigma_0/\varepsilon_0 \cos\phi \quad (17)$$

is called the storage modulus, and

$$E'' = \sigma_0/\varepsilon_0 \sin\phi \quad (18)$$

is the loss modulus. In an ideally elastic material, ϕ is equal to zero and σ/ε represents the elastic modulus, which obeys Hooke's law. However, most materials are anelastic, so ϕ is not zero and there exists a non-zero imaginary part of the complex modulus. The ratio of the loss modulus to the storage modulus is given by:

$$\eta = E''/E' = \tan\phi \quad (19)$$

where η is called the loss factor and $\tan\phi$ the loss tangent. Both η and ϕ are used to characterize the damping capacity of materials.

Specific damping capacity (SDC) is another common measure of damping. In the stress vs. strain curve a hysteresis loop is formed because of the lag of strain behind stress when the material is under cyclic loading. The area enclosed by the hysteresis loop represents the energy dissipated inside the material during one cycle. The magnitude of such a hysteresis at low stresses is representative of the damping capacity. Specific damping capacity (ψ), is the ratio of the dissipated energy (ΔW) during one complete cycle to the maximum stored energy (W) from the beginning of the loading to the maximum, is expressed as:

$$\psi = \Delta W/W \quad (20)$$

where

$$\Delta W = \int(\text{circle}) \sigma \, d\varepsilon \quad (21)$$

$$W = \int \sigma \, d\varepsilon \quad (\omega t: 0 \rightarrow \pi/2) \quad (22)$$

Where σ and ε are given by Eqs. (14) and (15).

Material damping is characterized not only by the phase lag of deformation behind the applied load in forced vibration and the suppression of resonant amplitude, but also by the decay of vibration amplitude in free vibration. There are also several other quantities, which can be used to characterize damping capacity. Among them, logarithmic decrement is a common measure of damping and is derived from the amplitude decay of the specimen under free vibration. Given the amplitude-time curve. The damping capacity at any stress may be quickly and accurately determined [4]. The logarithmic decrement, δ , is given by:

$$\delta = 1/n \ln(A_i/A_{i+n}) \quad (23)$$

where A_i and A_{i+n} are the amplitudes of the i th cycle and the $(i+n)$ th cycle, at times t_1 and t_2 respectively, separated by n periods of oscillation. A vibrating anelastic material is analogous to the Kelvin-Voigt model, which consists of a linear mechanical system with a spring (restoring force), and a bulk mass and a dashpot resistance (damping).

The inverse quality factor Q^{-1} , is another widely used material damping measure. It is defined by:

$$Q^{-1} = f_2 - f_1 / f_r \quad (24)$$

Where f_1 and f_2 refer to half power bandwidth frequencies and f_r is the resonant frequency in the spectrum of square amplitude vs. frequency for a specimen under forced vibration. The broadness of the resonant peak characterizes the magnitude of material damping.

This method has the advantage that the damping capacity can be measured accurately for very low stresses in the material. There are two objections, which can be raised in connection with this method [4]:

1. Since the magnitude of the stress cycle is continuously changing, it is not possible to study the variation in damping capacity with stress. Since stress appears to be one of the most significant variables involved, this represents a serious limitation.
2. The relationship between the shape of the resonance curve and the true damping capacity of the material depend on the type of relationship, which exists between stress and damping capacity. Since this stress-damping capacity relationship is not know in general for a particular material, there is some question as to the physical interpretation of the resonance curve.

For the cases of relatively small damping capacity (e.g. $\tan\phi < 0.1$), all the aforementioned damping quantities, ϕ , $\tan\phi$, η , Q^{-1} , δ and Ψ are interchangeable and are related by the following equation:

$$\phi = \tan\phi = \eta = Q^{-1} = \delta/\pi = \Psi/2\pi \quad (25)$$

Damping mechanisms

The damping behavior in crystalline materials may typically be attributed to several mechanisms, which include thermoelastic damping, magnetic damping, viscous damping and defect damping.

The first three types of damping mechanism generally result from the bulk response of a material the fourth mechanism, defect damping, is an intrinsic source and stems from the internal friction exerted on atomic movement in the regions of defects in crystalline metals and alloys. Among the damping mechanisms that are important in metals and alloys, defect damping represents a large part of the overall damping and is thought to be one main contributor to the overall damping of crystalline materials under conventional conditions. Therefore, both internal friction and damping are interchangeable terms in describing damping behavior for most of crystalline materials. According to defectology any type of defect will be a source to dissipate energy because of internal friction by the intrinsic movement of the defect under cyclic applied stress. The defects in polycrystalline materials and alloys include point defects (vacancies and disorders), line defects (dislocations), surface defects (grain boundaries and interfaces) and bulk defects (micropores and microcracks). The dislocations contribute to damping by the internal friction between the vibrating dislocation lines and their neighboring regions, the grain boundaries by their viscous sliding, the interface by the mobility of the incoherent microstructure at the strongly bonded interfaces, and the slip at the weakly bonded interfaces, and the micropores by the stress concentration and mode conversion around the pores.

To some extent these defects alter the otherwise perfect symmetry and uniformity of the metallic crystal structure. It is the asymmetry and non-uniformity that govern microstructural deformation behavior in a crystalline material with defects under applied load, and therefore dominate the macro-mechanical behavior of the material in service. In a perfect single crystal under an applied stress well below the yield stress, every unit cell of the crystal deforms uniformly and can restore its original position by interatomic bonding elasticity upon removal of the applied load. In an imperfect crystal, however, each defect causes distortion of the crystal symmetry and therefore induces an accompanying stress concentration. Although the magnitude of the externally applied stress is small, the internal stress concentration in some local regions may be sufficient to cause atomic rearrangement in regions of high density of defects. During the rearrangement, the atoms in the distorted region are capable of sliding by one another, which are viscous-like in nature in response to the applied load. At ambient temperatures, the relative atomic displacements are typically fractions of an atomic diameter. At high temperatures this sliding can be extensive and lead to viscoelastic strain. The displaced atoms introduce internal stress relaxation and the displacements can be recovered upon load removal. The internal friction is associated with the reversible or irreversible movements of atoms in the vicinity of defects under cyclic loading and leads to mechanical hysteresis in stress vs. strain behavior.

Damping mechanisms associated with secondary phases

The presence of secondary phases introduces a number of changes in the microstructure in terms of interfaces and dislocations and hence leads to a change in the overall damping behavior of metals and alloys. The secondary phases can be introduced by precipitation reaction-induced phase transformation or dispersion. Precipitation phenomena typically occur by ageing a metastable supersaturated solid solution alloy that has been solution treated and quenched from elevated temperatures.

Secondary phases influence damping behavior of metals and alloys by four possible mechanisms. First, the interfaces between the secondary phases and matrix dissipate energy under cyclic loading. Second, an enhanced dislocation density due to the thermal mismatch difference between secondary phases and matrix increases energy dissipation sources. Third, interactions between precipitates and dislocations or grain boundaries may lead to a change in damping response. Fourth, the precipitated secondary phase particles or added ceramic particles may possess

different intrinsic damping from that of the matrix material and thereby lead to a rule of mixtures effect on the overall damping behavior.

PM components are not precipitation hardened but solution hardened. The difference is that the atoms are well dispersed in the lattice instead of being present in clusters and as a consequence the damping effects will not be as pronounced.

Damping capacity

The work of numerous investigators, has established some facts about the general behavior of damping capacity which will be summarized here [4]:

1. The damping capacity is independent on frequency over a large range.
2. The damping capacity is dependent on the magnitude of the stress. In general the damping capacity increases with stress, although in some cases the opposite occurs. There is evidence to support that for very small stresses the damping capacity is essentially constant, but is still an appreciable amount.
3. The damping capacity increases with temperature. It has been suggested by Brophy that an apparent rise in damping capacity at high frequencies may be a temperature effect, since heat is being generated in the specimen by internal friction faster than it can be dissipated by the supports.
4. Some of the previous work indicates an increase of damping capacity with an increase in grain size, although this was not shown for all metals.
5. Very slight changes in the structure, composition, state and previous history of the specimen may have large effects on the damping capacity.

Considering the dependence of the damping capacity of most metals and alloys on strain amplitude, temperature and frequency, certain experimental conditions should be specified for the damping characterization.

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4. D. E. Hudson, "Internal Friction in Metals"
<http://resolver.caltech.edu/CaltechETD:etd-05232003-090926>

Recommended reading

1. M³D: Mechanics and Mechanisms of Material Damping, Eds.: A. Wolfenden and V.K. Kinra, ASTM STP 1169, American Society for Testing and Materials, Philadelphia, (1992)
“STP 1169 is a comprehensive summary of the state-of-the-science in damping measurements and their interpretation. This volume has valuable contributions from the leading practitioners in the field and for the first time brings together two diverse technical communities, namely, mechanics of solids and materials science as they concern the field of material damping.”

2. M³D III: Mechanics and Mechanisms of Material Damping, Eds.: A. Wolfenden and V.K. Kinra, ASTM STP 1304, American Society for Testing and Materials, Philadelphia, (1997)
“STP 1304 contains the latest research results on the fundamentals and applications of techniques for measuring and interpreting damping in all types of solids. The world’s leading experts in the field have presented 28 comprehensive papers covering various aspects of damping, including both fundamental research and technological applications. Topics explored include: Nonlinear effects of boundaries; • Viscoelastic damping; • Grain boundary damping; • High temperature damping; • Ceramics; • Martensitic and solid-liquid phase changes; • Magnetomechanical damping; • Elastothermodynamic damping; • Layered and intercalated materials; • Modeling; and • Longitudinal and flexural vibrations.”