Multi-Tachometer Order Tracking and Operating Shape Extraction

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Abstract

An automobile and a tracked military vehicle were instrumented with multiple tachometers, one for each drive wheel/sprocket and operated with accelerometers mounted at suspension, chassis, and powertrain locations on the vehicles. The TVDFT order tracking method was then used to extract the order tracks from each of the wheels/sprockets and operating shapes estimated based on the order tracks. It is shown that under some conditions a different operating shape is excited by each of the wheels/sprockets simultaneously. This is due to the asymmetries present in the vehicles. The strengths of the TVDFT order tracking method are also shown for this type of analysis which is difficult due to the closeness and crossing of the orders generated by each of the wheels. Benefits of using multiple tachometers and advanced order tracking methods becomes apparent for solving certain types of noise and vibration problems.

1 Introduction

Two different vehicles were instrumented with accelerometers in many different locations, both near the dominant vibration sources and at user interface locations. The vehicles were also instrumented with multiple tachometer sensors and operated under various operating conditions. The tachometer data was then used to track the orders caused by the instrumented components.

The first vehicle analyzed was a front wheel drive automobile operating on a chassis dynamometer with tachometer signals measured on each of the front wheels. A slight tire inflation difference was introduced into the tires to give them different apparent diameters so that the frequencies of the tire unbalance would be slightly different at any point in time. These two orders are very close to another, less than 1 rpm apart in most instances. Due to the closeness of these orders it is shown that the contributions from the two tires are very difficult to separate. The TVDFT with a post-processing technique is used to attempt to separate these orders.

The second vehicle instrumented and analyzed was a tracked military vehicle which had both of its drive sprockets instrumented to measure their rotational speeds as well as accelerometers at many different response locations. This vehicle was then operated on a test track with many turns. Due to the difference in the track speeds, when each corner is traversed it is possible to separate the contributions between the left and right track engagement.

In both of the vehicle cases analytical data was used with the experimentally measured tachometer signals to verify the performance of the order tracking algorithms and to lend the user confidence that the algorithms were performing as expected.

Finally, recommendations are made on the performance and use of the TVDFT to separate very close orders when multiple tachometer signals are measured on operating machinery.

2 Time Variant Discrete Fourier Transform Order Tracking Theory

The time variant discrete Fourier transform (TVDFT) method of order tracking is a special case of the chirp-z transform. The chirp-z transform is defined as a type of Fourier transform with a kernel whose frequency and damping vary as a function of time [1]. The TVDFT is defined as a discrete Fourier transform whose kernel varies as a function of time defined by the rpm of the machine, but the damping does not vary as a function of time. The TVDFT has many of the advantages of the resampling based order tracking methods, while reducing the computational load of the calculations considerably [2,3].

The TVDFT method is based on constant delta-t sampled data. Whether it is desired to analyze data with a constant frequency or constant order bandwidth determines whether the sampling theorem used is based on constant Δt data or constant $\Delta\theta$ data.

The TVDFT is based on the transform shown in Equation 1. It should be noted that the kernel of this transform appears as a portion of the structure equation used in the order tracking Kalman filter [4]. This kernel is a cosine or sine function of unity amplitude with an instantaneous frequency matching that of the tracked order at each instant in time. This kernel may also be formulated in a complex exponential format similar to the corresponding Fourier transform.

$$\begin{aligned} a_n &= \frac{1}{N}\sum_{n=1}^{N} x(n\Delta t)\cos\left(2\pi\int_0^{n\Delta t}(o_n * \Delta t * rpm/60)dt\right) \\ b_n &= \frac{1}{N}\sum_{n=1}^{N} x(n\Delta t)\sin\left(2\pi\int_0^{n\Delta t}(o_n * \Delta t * rpm/60)dt\right) \end{aligned} \tag{1}$$

where: o_n is the order which is being analyzed.

a_n is the Fourier coefficient of the cosine term for o_n.

b_n is the Fourier coefficient of the sine term for o_n.

rpm is the instantaneous rpm of the machine.

This transform is best suited to estimate an order with a constant order bandwidth. A constant order bandwidth estimate may be obtained by performing the transform over the number of time points required to achieve the desired order resolution, where the order resolution is defined as the inverse of the number of revolutions in the analysis block. This implies that as the rpm increases, the transform will be applied over a shorter time, giving a wider Δf equivalent to a constant order bandwidth. This behavior is also exhibited by the resampling methods and is advantageous for order tracking. This transform is normally only performed for the orders that are desired and not for a full spectrum.

Since the frequency of the kernel of this transform matches the frequency of the order of interest at each instant in time, there is no leakage due to the order not falling on a spectral line. There will, however, be leakage effects from other orders that are present in the data. These orders can “leak” into the frequency band of analysis around the order. Typically used windows for conventional FFT analysis are also used with this transform. Since all windows have a frequency resolution/amplitude estimate tradeoff, the window chosen can have a significant effect on the results. Which window to use depends on the order content of the data and the aspects of the order estimate the user feels are most important.

The TVDFT order tracking method presented here is a very practical order tracking method which can be implemented in a very efficient manner on a computer. This method contains many of the advantages of the resampling based algorithms without much of the computational load and complexity. Computational efficiency is gained for large numbers of channels by computing the transform kernel once, storing it, then applying it to each channel. Any window used in the analysis should be applied to this pre-computed kernel, since the window only has to be applied once if it is applied to the kernel instead of once for each channel.

2.1 Orthogonality Compensation Matrix Theory

To enhance the capabilities of the TVDFT for tracking orders and to reduce the errors due to non-orthogonality of the kernels, an orthogonality compensation matrix (OCM) may be applied. The application of the OCM allows faster sweep rates to be analyzed, as well as closely spaced and crossing orders to be analyzed more accurately[5]. This OCM is applied as a post-processing of the order estimates from the TVDFT analysis.

To apply the OCM, all orders of interest are first tracked using the TVDFT. This tracking should be done intelligently, as the quality of the compensation is related to the quality of the original order estimates. This implies that the user may want to apply a Hanning window to increase out of band rejection over a Rectangular window. The bandwidth used may be somewhat wider than is minimally necessary to separate closely spaced orders. The amount of relaxation of the minimum bandwidth depends on the window used in the analysis. This relaxation of the bandwidth allows fewer revolutions to be analyzed at a time if desired, which allows faster sweep rates to be analyzed.

The application of the OCM is a linear equations formulation that is shown in Equation 2.

$$\begin{bmatrix} e_{11} & e_{12} & e_{13} & \cdots & e_{1m} \\ e_{21} & e_{22} & e_{23} & & \\ e_{31} & e_{32} & e_{33} & & \vdots \\ \vdots & & & \ddots & \\ e_{m1} & & \cdots & & e_{mm} \end{bmatrix} \begin{Bmatrix} o_1 \\ o_2 \\ o_3 \\ \vdots \\ o_m \end{Bmatrix} = \begin{Bmatrix} \tilde{o}_1 \\ \tilde{o}_2 \\ \tilde{o}_3 \\ \vdots \\ \tilde{o}_m \end{Bmatrix} \tag{2}$$

where: e_{ij} is the cross orthogonality contribution of order *i* in the estimate of order *j*.

o_i is the compensated value of order *i*.

$\tilde{o}_i$ is the estimated value of order *i* obtained using the TVDFT.

The cross orthogonality terms, e_{ij}, are calculated by applying the kernel of order *i* to the kernel of order *j*, as shown in Equation 3.

$$e_{ij} = \frac{1}{N}\sum_{n=1}^{N}\left\{\exp\left(2\pi\int_{0}^{n\Delta t}\left(o_i * \Delta t * rpm/60\right)dt\right) \times Window\right\} \times \exp\left(2\pi\int_{0}^{n\Delta t}\left(o_j * \Delta t * rpm/60\right)dt\right)^{*} \tag{3}$$

The window used in the original order estimate is applied to order *i* to compensate for any correction factor that may need to be applied to scale the data correctly. It also includes the effects of the shape of the window in the compensation. Each term in the matrix represents the amount that the orders' kernels interact with one another in the transform estimation. If the orders included in the calculation of the OCM are orthogonal, the off diagonal terms of this matrix will be zero, as is the case for the standard Fourier transform kernels. Since the effects of any orders not included in this calculation are not compensated, it is recommended that all significant orders be included in the compensation calculation.

Very closely coupled orders are normally very difficult to separate using standard FFT or resampling techniques, as well as the TVDFT without compensation, because the orders may beat with one another. However, with compensation the TVDFT can separate the contributions of the orders effectively. Initially, the orders should be tracked with a bandwidth that is at its largest approximately equal to the spacing of the closely coupled orders. If the orders are tracked with this bandwidth using a Hanning window, the order estimates will contain beating of the two orders. This beating effect can be removed by applying OCM.

Crossing orders pose a similar problem to that of closely spaced orders. Oftentimes, if two orders cross one another, the order estimates are incorrect at the crossing rpm due to the interaction of the orders. Tracking the orders and then applying the OCM allows the separation of the contributions from each order.

3 Automobile Analysis

3.1 Analytical Example Based on Experimental Tachometer Signals

An experimental dataset was acquired by instrumenting a front wheel drive vehicle with accelerometers on the front suspension spindles, powertrain, and powertrain cradle. Tachometer signals were measured on both of the front wheels using ICP LaserTach's. This vehicle was then operated on a chassis dynamometer with a difference in air pressure between the two tires of 2 psi. This air pressure difference was introduced to force the two wheels to spin at different speeds since the chassis dynamometer was not able to vary the wheel speeds independently. Each of the tires had a 2 gram unbalance weight installed to ensure that there was a high enough unbalance force to excite modes of the vehicle.

A speed sweep was then performed from approximately 80 kph up to 110 kph. The tachometer signals were then analyzed to obtain a course estimate of each of the wheel's speeds in rpm which was then curve fit using a spline fit to obtain a smooth rpm vs. time function. This curve fitting is justified due to the inertia of the vehicles wheels and the fact that it is known that the dynamometer was programmed to give a smooth speed sweep. The variation on the tachometer signals is due to the inaccurate estimate of the instantaneous rotational speed because of the finite sampling speed used to capture the tachometer pulse train. Both the raw and the curve fit tachometer signals are displayed in Figure 1 below. Note how close in rpm the left and right wheels are in the zoom in of only the curve fit signals in the right plot.

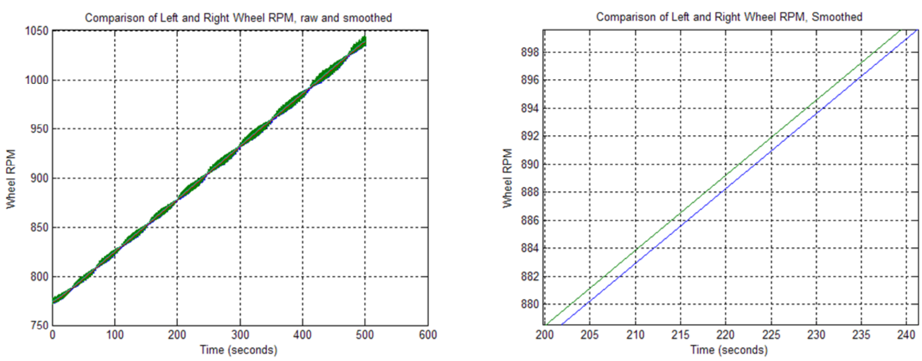


Figure 1: Right plot: Raw and curve fit RPM signals. Left plot: Zoomed in view of left and right tachometer signals.

Upon having obtained a smooth rpm function for each wheel these tachometer signals were used to generate a dataset of constant amplitude sinusoidal functions whose frequencies exactly match those of the estimated rpm at all points in time. This means that the sinusoidal functions are actually representations of the first order for each of the wheels. These two functions were then summed together to create one virtual response. This virtual response is necessary to evaluate the performance of various order tracking variables. Without these analytical signals it would be impossible to evaluate whether the results of this order tracking were accurate.

The first analytical signal was created with amplitude of 1 for the left tire and amplitude of 0.01 for the right tire. This test case was created with different amplitudes to evaluate whether the order tracking algorithms could separate the two responses from one another when one amplitude was 100 times larger than the other amplitude. This analytical signal was then used as an input to the TVDFT order tracking algorithm to assess its performance.

An attempt was also made to use the 1st, 2nd, and 3rd order Vold-Kalman filters to track these analytical orders. None of the filters was capable of performing this analysis as the orders are only 0.0167 Hz apart at 800 rpm. While these filters can have very sharp passband characteristics the coupling of the frequency separation and difference in amplitudes by a factor of 100 proved too difficult a dataset for these filters. However, it is believed that the Vold-Kalman filters with un-coupling may have been able to separate these two orders, this was not attempted as the author does not have the un-coupling implemented at this time.

3.1.1 TVDFT with Orthogonality Compensation

The TVDFT with the orthogonality compensation is sensitive to the condition number of the cross matrix since this matrix must be inverted. For this reason, numerous order resolutions were investigated, the goal is to use the largest order resolution possible and still get good separation of the orders. Larger order resolutions are obtained by performing the transform over fewer cycles with therefore means that faster amplitude variations can be accurately tracked.

The condition number vs. rpm and order amplitudes vs. rpm are given in Figure 2 for an order resolution of 0.2 orders. It can be seen that the condition number is relatively high, on the order of 10^5, and that the order tracks are not very accurate due to this condition number resulting in an inaccurate matrix inverse.

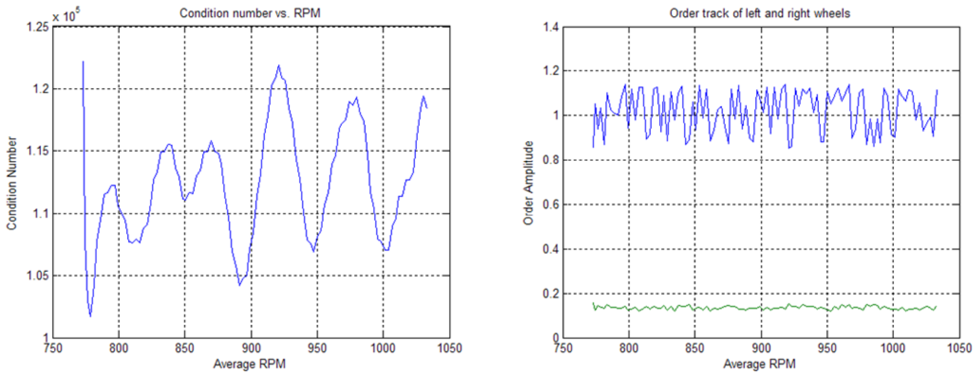


Figure 2: Condition number and TVDFT order track results with 0.2 order resolution.

Using an order resolution of 0.1 produces the condition number and order tracks shown in Figure 3 below. It can be seen that the quality of these order tracks is much better than those obtained using a resolution of 0.2 orders. With an order resolution of 0.1 the condition number is on the order of 10^4, this leads to a much better order estimate however zooming into the order plot it is seen that the order track of the right wheel with an amplitude of 0.01 is still not very accurate. This is shown in Figure 4.

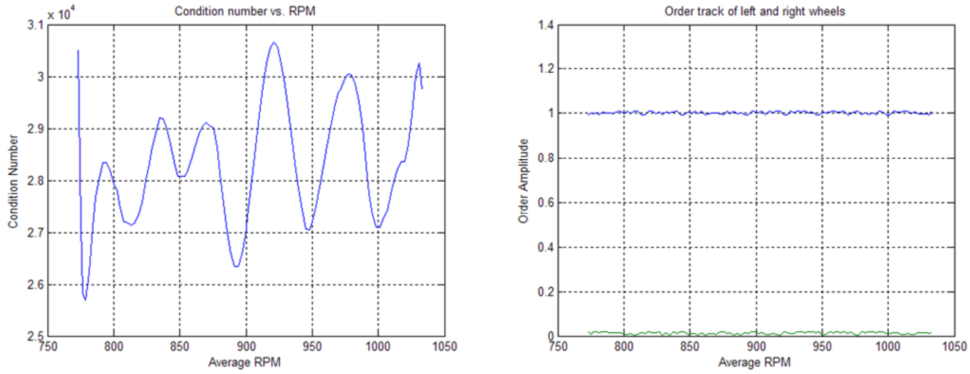


Figure 3: Condition number and TVDFT order track results with 0.1 order resolution.

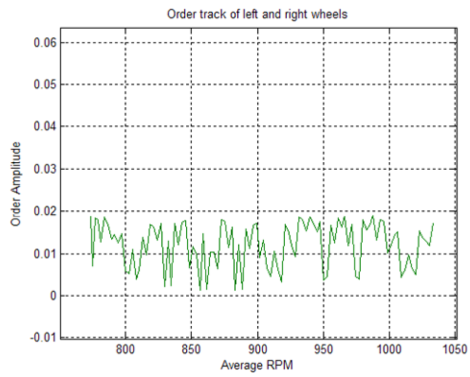


Figure 4: Zoom in of order track, theoretical amplitude = 0.01.

The order resolution was then changed to 0.05 orders and the order track repeated. The results of this order track are shown in Figure 5. Note that the condition number is now on the order of 6000 to 7000. With this condition number even the zoom in of the right wheel with an amplitude of 0.01 is quite accurate.

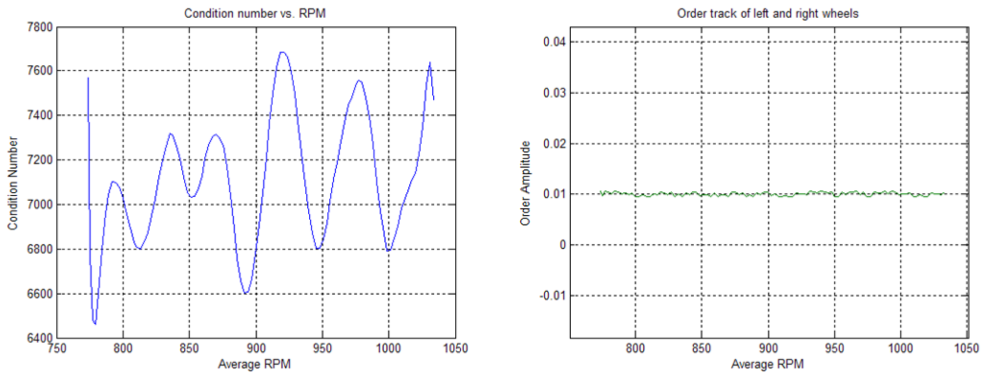


Figure 5: Condition number and TVDFT order track results with 0.1 order resolution.

Having determined that the order resolution of 0.05 provides very accurate order tracks with the measured tachometer signals and the TVDFT algorithm the actual vehicle operating data can be analyzed.

3.2 Experimental Data

Using the 0.05 order resolution determined in the previous section to be appropriate for the measured tachometer signals and still be able to separate left/right contributions with a difference in amplitude of a factor of 100 the experimentally acquired vehicle data was processed.

Figure 6 below shows the results of tracking the 1st order tire imbalance from both tires at two different measured degrees of freedom, the left side knuckle and the right side knuckle. It can be seen that at these two locations the contributions from the left and right tires are considerably different from on another as would be expected. It can also be seen that the order tracks are not very smooth which is not expected.

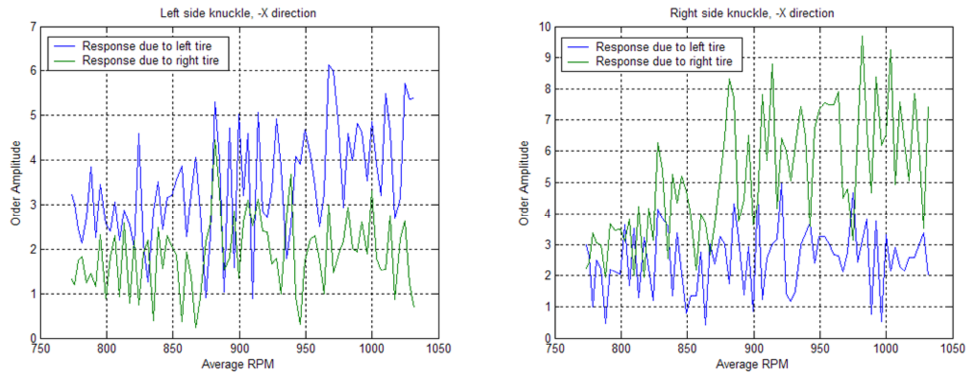


Figure 6: Experimental order tracks with order resolution of 0.05.

In an effort to obtain smoother order estimates the order resolution was changed to 0.01, this higher resolution implies that the data is averaged over 100 revolutions to compute the order estimates as opposed to the 20 revolutions of the previous analysis. Figure 7 shows the same two degrees of freedom with this new resolution. It is clearly observed how increasing the average time has smoothed the order estimates.

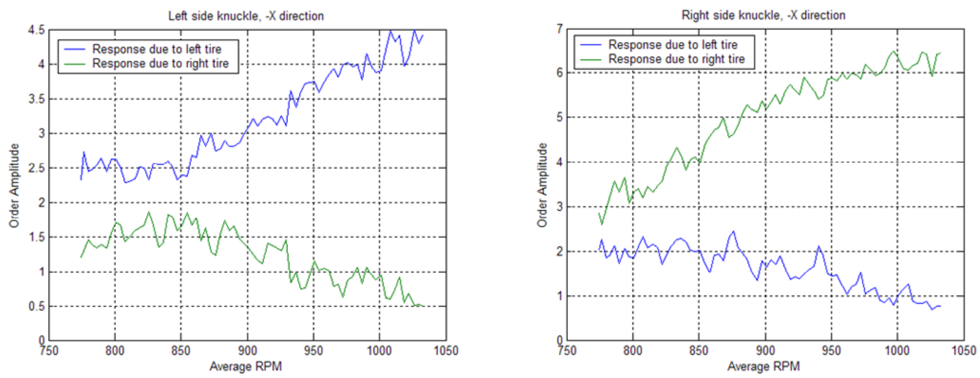


Figure 7: Experimental order tracks with order resolution of 0.01.

To fully understand the differences in the responses from the left and right side tires order tracks were done of all measured degrees of freedom and operating shapes estimated from these order tracks. Figure 8 shows the left and right wheel excited shapes at an rpm of 901. Note how at this rpm the left tire excites primarily a fore/aft translational mode while the right tire excites primarily a torsional mode where the left tire is nearly a node.

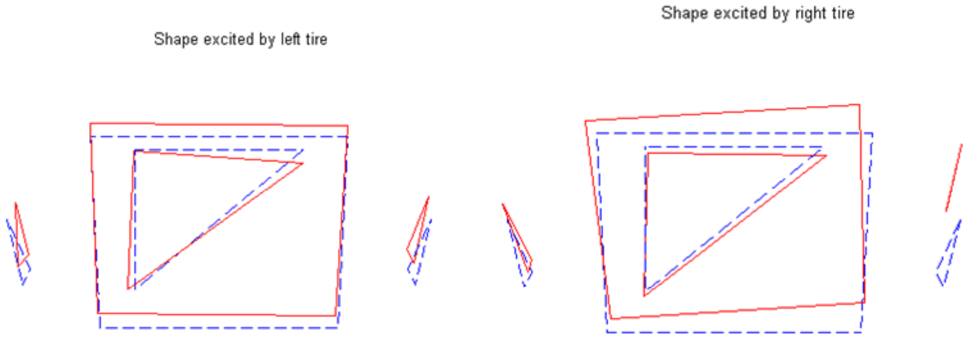


Figure 8: Left and Right wheel excited operating shapes at 901 rpm.

A second set of operating shapes is shown in Figure 9, these shapes were estimated at 1015 rpm. In this set of shapes, both shapes are essentially torsional in nature with the opposite wheel of the excitation wheel acting as a node.

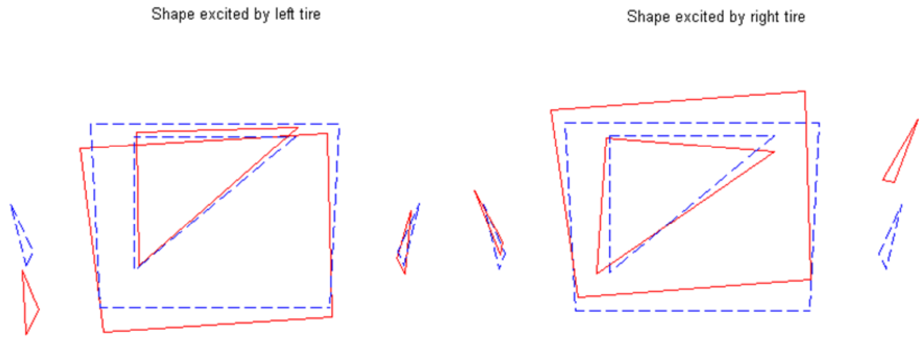


Figure 9: Left and Right wheel excited operating shapes at 1015 rpm.

In conclusion, from this set of experimental data it can be seen that there indeed can be benefits in the ability to separate the left and right wheel excitations from one another on an automobile.

4 Tracked Military Vehicle Analysis

Data was acquired on a tracked military vehicle where each of the drive sprockets were instrumented with encoders to measure their rotational speeds. There were accelerometers installed at various locations around the vehicle including several passenger interface locations, suspension locations, and electronic instruments. This fully instrumented vehicle was then driven around an off-road course that had many combinations of turns and straight-aways. Data was acquired in various locations on this track and is analyzed the following sections to understand whether various locations are excited predominantly by the left or right track system or whether they experience nearly equal excitation from both track systems.

The approach to analyze this data is the same approach taken in the automotive example where the first step is to work with the measured tachometer signals and synthesize a response signal to understand the performance of the order tracking algorithms.

4.1 Analytical Example Based on Experimental Tachometer Signals

The first dataset chosen to analyze from the off-road course has a sharp turn and some gradual turning present in it. The first step in analyzing this data was to process the tachometer signals. The tachometer signals were measured using encoders which caused them to have a high amount of variance, to reduce the variance the measured tachometer signals were fit with a spline fitting algorithm so that they were smoothly varying functions of time. Based on the smoothed tachometer signals the condition number can be calculated for the matrix of order tracking kernels. This matrix should include the kernels of all orders of interest, for this analysis it included 1^{st}, 2^{nd}, and the first three sprocket meshing orders. The smoothed RPM and the condition number of the compensation matrix are shown below in Figure 10.

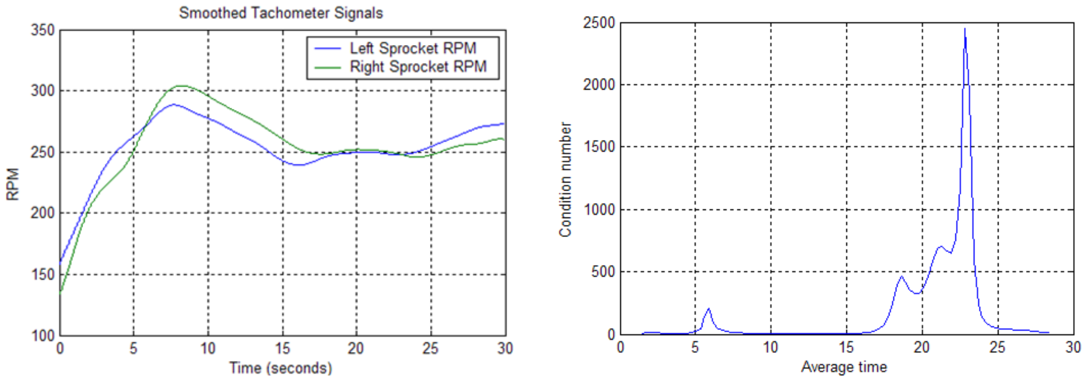


Figure 10: Smoothed tachometer signals and condition number for 1^{st} off-road course operating condition.

The condition number in Figure 10 was calculated for an order resolution of 0.1 orders. Using the smoothed tachometer signals, analytical response signals were generated with the left sprocket's response having an amplitude of 0.01 and the right sprocket's response having an amplitude of 1. The sprocket meshing orders from both sprockets were then tracked, in this example the orders for the left sprocket were tracked using both the uncompensated and the orthogonality compensated TVDFT algorithms. The uncompensated order amplitude was only estimated for the 0.01 amplitude response as this is where the coupling is most visible. Figure 11 shows the results of this order tracking on the analytical data.

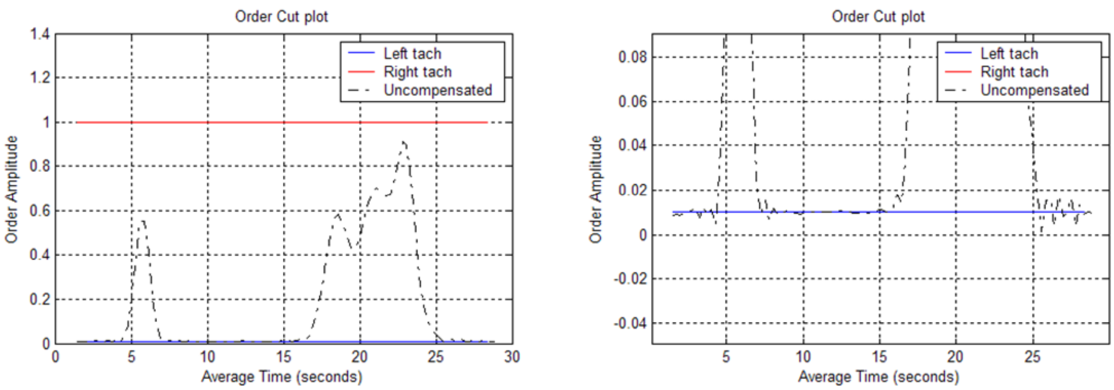


Figure 11: Uncompensated and Compensated order tracks of analytical data with amplitudes 1.0 and 0.01. Left plot is a zoom in view of 0.01 amplitude.

As can be seen in Figure 12 the uncompensated TVDFT does not accurately estimate the amplitude of the 0.01 amplitude response where the two tachometer signals cross one another or are close to another in rpm. However, the compensated TVDFT accurately separates the orders at these locations.

The same analysis was done on a second operating condition from the off-road course. The tachometer signals and condition numbers are shown in Figure 12.

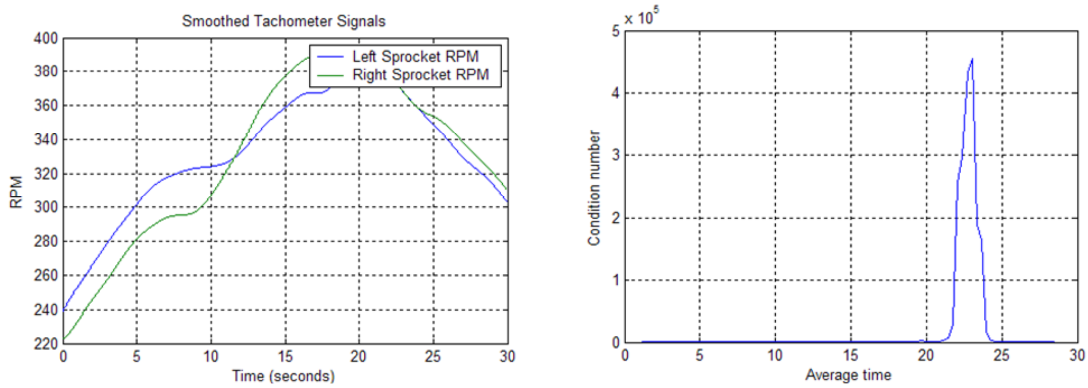


Figure 12: Smoothed tachometer signals and condition number for 2nd off-road course operating condition.

Using the smoothed tachometer signals analytical signals were again generated and tracked. These results can be seen in Figure 13. Again, it can be seen that the uncompensated order estimate from the TVDFT is in error when the two tachometer signals cross one another.

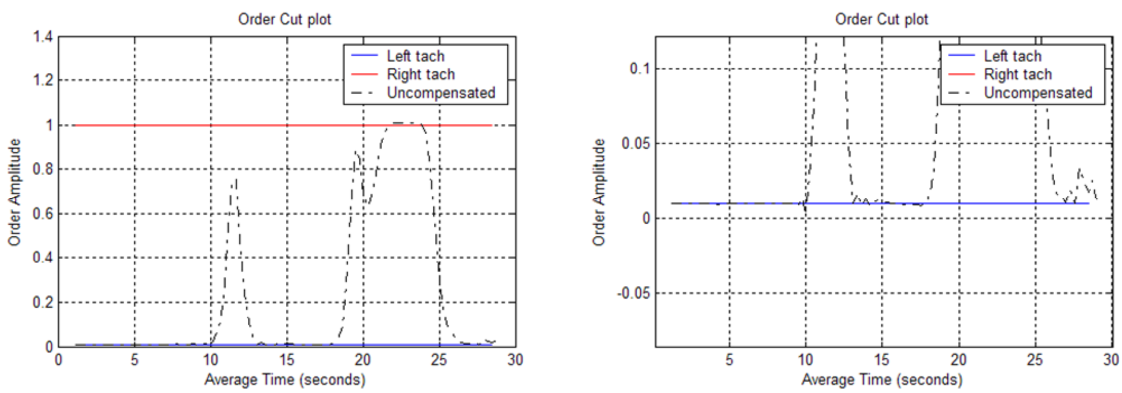
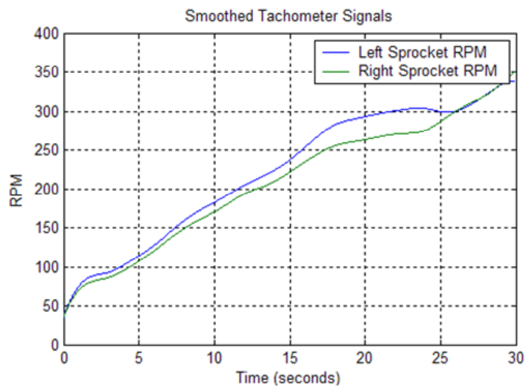


Figure 13: Uncompensated and Compensated order tracks of analytical data with amplitudes 1.0 and 0.01. Left plot is a zoom in view of 0.01 amplitude.

Finally, a third operating condition was also chosen to analyze from this set of tests. This operating condition has the two sprocket rpm's close to one another for a large portion of the dataset. Figure 14 shows the tachometer signals and the condition number associated with this dataset. It can be seen that this dataset is a broad corner to the right.



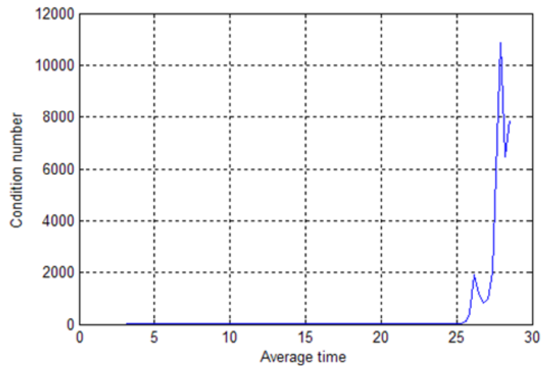
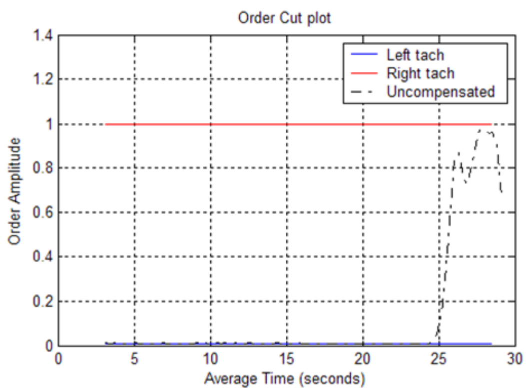


Figure 14: Smoothed tachometer signals and condition number for 3rd off-road course operating condition.

Figure 15 shows the performance of the TVDFT on this dataset. In this dataset the uncompensated estimate is accurate up until approximately 25 seconds which is where the two rpm signals become very close in frequency, at this point the uncompensated TVDFT loses its accuracy.



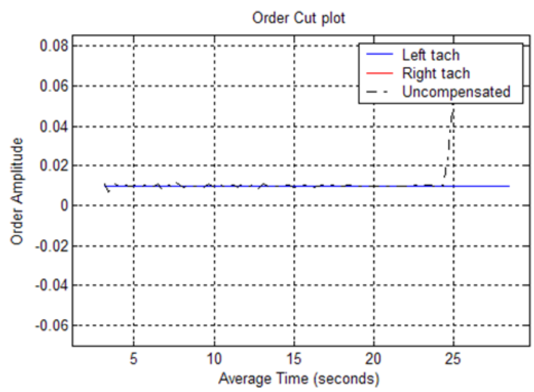


Figure 15: Uncompensated and Compensated order tracks of analytical data with amplitudes 1.0 and 0.01. Left plot is a zoom in view of 0.01 amplitude.

4.2 Experimental Data

Having confidence that the compensated TVDFT is accurately estimating the amplitudes of the left and right sprocket responses and separating their contributions when the rpm of the two sprockets is close or the rpm cross one another the actual data was analyzed for these three datasets.

The first dataset was analyzed at two locations, one location being a suspension location on the left side and the other location being a user interface location. It is expected that the left side suspension location be dominated by the left side sprocket excitation and that the user interface location have more similar contributions from the left and right side sprocket engagements. The results of this analysis are shown in Figure 16. As expected in the suspension results the left side sprocket dominates the response. Interestingly in the user interface location results the left and right side each dominate at different portions of the operating condition.

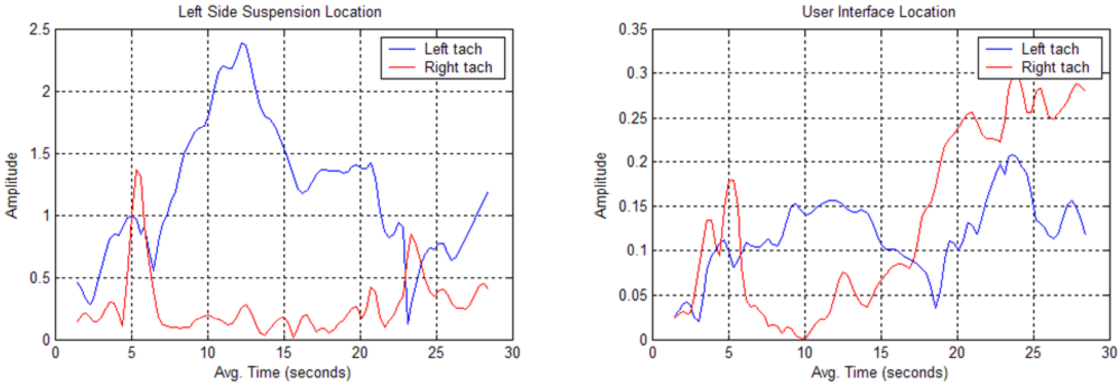


Figure 16: Left side suspension location and user interface location, operating condition 1.

Operating condition 2 was analyzed at a right suspension location and a user interface location. These results are shown in Figure 17.

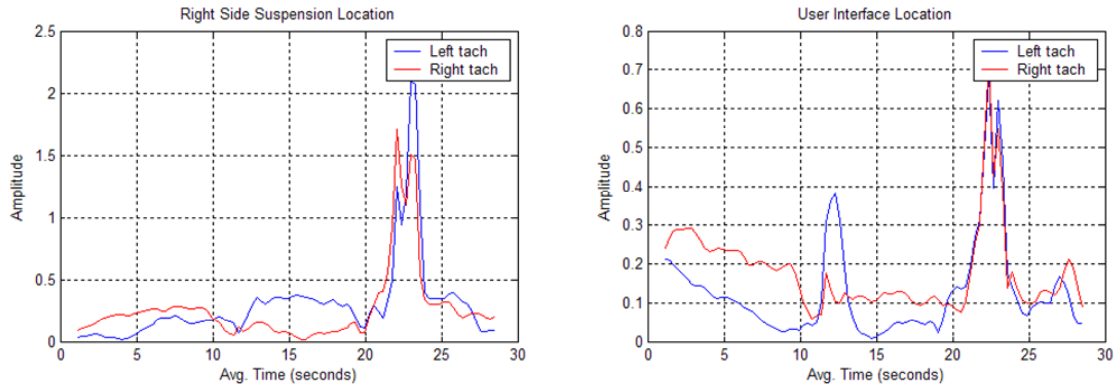


Figure 17: Right side suspension location and user interface location, operating condition 2 with 0.1 order resolution.

In Figure 17 it is seen that the TVDFT is not performing as expected since in both locations the responses appear to have equal contributions from both sprockets at the same time. This phenomena was also seen in the automotive data and can be attributed to the condition number of compensation matrix being too large and indicating the matrix inverse may be inaccurate. This condition number can be seen in Figure 12 and approaches 10^5. In an attempt to more accurately analyze this data the order resolution was changed from 0.1 to 0.02 and the data re-processed. These reprocessed results can be seen in Figure 18.

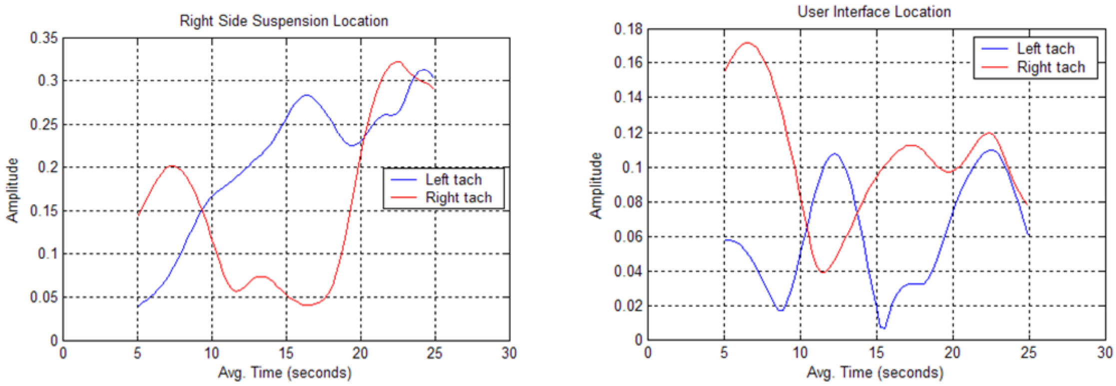


Figure 18: Right side suspension location and user interface location, operating condition 2 with 0.02 order resolution.

The results of Figure 18 on the right side suspension location are particularly interesting in that at some points in time the left side excitation actually dominates the response. The user interface location again varies at each point in time as to what is the dominant excitation as expected. It should also be noted how smooth the order estimates are with an order resolution of 0.02 orders, this is because the higher resolution uses much more data to estimate the amplitudes and hence averages many of the fluctuations out of the data.

Finally the dataset from operating condition 3 was processed to assess whether the order tracking algorithms performed as expected on this dataset. The results of this analysis are shown in Figure 19.

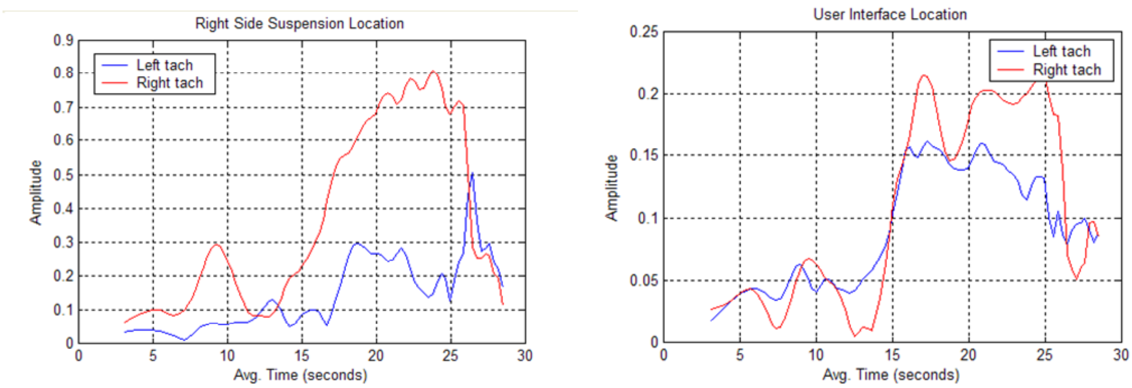


Figure 19: Right side suspension location and user interface location, operating condition 3 with 0.1 order resolution.

The estimated amplitudes for the right side suspension location are for the most part what would be expected but it is still interesting that for a short period of time the left side excitation dominates this response. The user interface location again shows that the dominant source changes as a function of time but is similar in amplitude between the two sides the majority of the time.

5 Conclusions

A brief summary of the time Variant Discrete Order Tracking, TVDFT, method was presented, this algorithm was then used to process data acquired both on an automobile operating on a chassis dynamometer and on data acquired on a tracked military vehicle operating on an off-road test course.

These datasets contained many instances of the primary excitation from the left and right sides being either very close in frequency or crossing one another in frequency. These datasets were used to evaluate the performance of the TVDFT with orthogonality compensation in effectively separating the contributions from left and right sides. To assess the performance on data with a known solution the experimentally measured tachometer signals were used to generate analytical datasets. These analytical datasets were then analyzed to get an understanding as to what order tracking resolution was required to obtain accurate order tracking estimates. Once the proper order tracking resolution was determined the same analysis was repeated on the experimental data.

In most cases the experimental analysis gave the results expected, however, in two instances the order tracking resolution determined with the analytical data was not high enough to effectively process the experimental data. Upon examination of the condition number of the compensation matrix, it became apparent that the matrix inverse required was not accurate due to high condition numbers. Increasing the order resolution lowered the condition number and produced much more believable results. It is thought that the discrepancies between the analytical data processing and the experimental data processing stem from fact that the experimental data has noise, other orders present, and changes in amplitude as a function of time whereas the analytical data only the had orders tracked in it, had no noise, and had response amplitudes that did not vary as a function of time.

Interesting results that were obtained included the estimation of different operating shapes due the left and right wheel excitations on the automobile as well as an understanding that as a function of time the dominant contributor to several different response locations varied between the left and right excitation sources on both the automobile and the tracked military vehicle.

Overall, a better understanding as to the capabilities and limitations of the ability of the TVDFT to track and separate close/crossing orders was obtained on experimental data.

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