Dr. Peter Avitabile
Structural Dynamics & Acoustic Systems Lab

Shaker Excitation

Peter Avitabile
UMASS Lowell

Marco Peres
The Modal Shop
**Shaker Excitation**

**Objectives of this lecture:**

- **Overview some shaker excitation techniques commonly employed in modal testing**
- **Review deterministic and non-deterministic methods**
- **Present excitation techniques that have developed from a historical standpoint**
- **Present some MIMO testing information**
Vibration Shaker Qualification vs Modal Shaker

Many people are familiar with vibration shakers used for qualification of equipment where specific loading is applied to replicate the actual operating environment.

This is a much different testing technique than what is done for modal testing (where high loads are not applied to the structure).
Shaker Excitation for Modal Testing

Excitation device is attached to the structure using a long rod called a “stinger” or “quill”

Its purpose is to provide input along the shaker excitation axis with essentially no excitation of the other directions

It is also intended to be flexible enough to not provide any stiffness to the other directions

The force gage is always mounted on the structure side of the quill

NOT ON THE SHAKER SIDE
**Excitation Configuration**

**Shaker**

- Test Signal
  - random
  - burst Random
  - pseudo-random
  - periodic-random
  - Chirp

**Power Amplifier**

**Stinger**

**force sensor**

**structure**

AUTORANGING AVERAGING

AUTORANGING AVERAGING WITH WINDOW

1 2 3 4
Reason for Stinger

Purpose of Stinger

• Decouple shaker from test structure

• Force transducer between stinger and structure decouple forces acting in the axial direction only

• Forces acting in any other direction will be unaccounted for creating error in the measurements
Possible Problems with Stinger

- Suspect increase in stiffness when stinger is at higher location

Axial stiffness  Axial and bending stiffness
Stinger Configuration with Through Hole Shaker

- 2-part chuck assembly
- Force sensor
- collet
- armature
- stinger

Modal Exciter

Test Structure
Common Stingers

- **Piano wire**
- **Modal stinger**
- **Threaded metal rod**
- **Threaded nylon rod**
Common Stingers

Types of Stingers Available

- Drill Rod

- Threaded Rod
  - Metal
  - Nylon

- Piano Wire
  - Axial stiffness provided through a preload on wire
  - Essentially no lateral stiffness
  - Requires shaker and test fixture to be fixed
**Shaker Excitation**

**CORRECT**

Force gage "divorces" the stinger/shaker from the structure

**WRONG**

Stinger becomes part of the test structure
The Overall Measurement Process

**INPUT**
- INPUT FORCE
- WINDOWED INPUT
- AVERAGED INPUT POWER SPECTRUM
- FREQUENCY RESPONSE FUNCTION

**OUTPUT**
- OUTPUT RESPONSE
- WINDOWED OUTPUT
- AVERAGED CROSS POWER SPECTRUM
- COHERENCE FUNCTION

**WINDOWED SIGNAL**
- WINDOWED INPUT
- WINDOWED OUTPUT

**AVERAGED INPUT, OUTPUT AND CROSS SPECTRA**
- AVERAGED INPUT
- AVERAGED OUTPUT
- AVERAGED CROSS

**COMPUTED FREQUENCY RESPONSE FUNCTION AND COHERENCE**
Signal Types

Excitation techniques can be broken down into two categories:

- Deterministic Signals
- Non-Deterministic (Random) Signals
Signal Types - Deterministic

Deterministic Signals

- conform to a particular mathematical relationship
- can be described exactly at any instant in time
- response of the system can also be exactly defined if the system character is known
- swept sine, sine chirp, digital stepped sine are examples
Signal Types - Non-Deterministic

Non-Deterministic (Random) Signals
- do not conform to a particular mathematical relationship
- can not be described exactly at any instant in time
- described by some statistical character of the signal
- generally have varying amplitude, phase and frequency content at any point in time
- pure random, periodic random, burst random are examples
<table>
<thead>
<tr>
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<tr>
<td><strong>Deterministic Signals</strong></td>
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Excitation Signal Characteristics

RMS to Peak
Signal to Noise
Distortion
Test Time
Controlled Frequency Content
Controlled Amplitude Content
Removes Distortion Content
Characterizes Non Linearities
# Summary Excitation Signal Characteristics

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<tbody>
<tr>
<td>Minimize Leakage</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Signal-to-Noise Ratio</td>
<td>Very High</td>
<td>Fair</td>
<td>Fair</td>
<td>Fair</td>
<td>High</td>
<td>Low</td>
<td>Fair</td>
</tr>
<tr>
<td>RMS-to-Peak Ratio</td>
<td>High</td>
<td>Fair</td>
<td>Fair</td>
<td>Fair</td>
<td>High</td>
<td>Low</td>
<td>Fair</td>
</tr>
<tr>
<td>Test Measurement Time</td>
<td>Very Long</td>
<td>Good</td>
<td>Very Short</td>
<td>Fair</td>
<td>Fair</td>
<td>Very Short</td>
<td>Very Short</td>
</tr>
<tr>
<td>Controlled Frequency Content</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Controlled Amplitude Content</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
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<tr>
<td>Removes Distortion</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Characterize Nonlinearity</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
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</table>

* Special Hardware Required

Ref: University of Cincinnati
Remarks on General Excitation Characteristics

The complete solution of a forced harmonic excitation will result in two parts of the response:
- transient part which decays with time and
- the steady state part of the response

\[ x(t) = \frac{F_0}{k} \frac{\sin(\omega t - \phi)}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\left(\frac{\omega}{\omega_n}\right)\right)^2}} \]

\[ + X_1 e^{-\zeta \omega_n t} \sin\left(\sqrt{1 - \zeta^2} \omega_n t + \phi_1\right) \]

Steady State

Transient
Remarks on General Excitation Characteristics

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- the steady state part of the response

\[
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\]

\[
+ X_1 e^{-\zeta \omega_n t} \sin\left(\sqrt{1 - \zeta^2} \omega_n t + \phi_1\right)
\]

Steady State

Transient
Vibrations - Convolution for SDOF Sine Excitation

Start of Sine

Steady State Reached

End of Transient

Sine input AVI
Swept Sine Excitation

Slowly changing sine signal sweeping from one frequency to another frequency
Analog Slow Swept Sine Excitation

A slowly changing sine output sweeping from one frequency to another frequency

ADVANTAGES
- best peak to RMS level
- best signal to noise ratio
- good for nonlinear characterization
- widely accepted and understood

DISADVANTAGES
- slowest of all test methods
- leakage is a problem
- does not take advantage of speed of FFT process
Random Excitation

An ergodic, stationary signal with Gaussian probability distribution. Typically, has frequency content at all frequencies.
Random Excitation

An ergodic, stationary signal with Gaussian probability distribution. Typically, has frequency content at all frequencies.

ADVANTAGES

• gives a good linear approximation for a system with slight non-linearities
• relatively fast
• relatively good general purpose excitation

DISADVANTAGES

• leakage is a very serious problem
• FRFs are generally distorted due to leakage
**Random Excitation**

**Time signal**

**Frequency Signal**

Notice that the coherence is very poor at all frequencies
Random Excitation

Effects of averaging

CH1 Pwr Spec
0Hz 800Hz
AVG: 1
-40
-90
dB Mag

CH1 Pwr Spec
0Hz 800Hz
AVG: 10
-40
-90
dB Mag

CH1 Pwr Spec
0Hz 800Hz
AVG: 100
-40
-90
dB Mag
Random Excitation with Hanning Window

An ergodic, stationary signal with Gaussian probability distribution. Typically, has frequency content at all frequencies.
Random Excitation with Hanning Window

An ergodic, stationary signal with Gaussian probability distribution. Typically, has frequency content at all frequencies.

ADVANTAGES

• gives a good linear approximation for a system with slight non-linearities
• relatively fast
• overlap processing can be used
• relatively good general purpose excitation

DISADVANTAGES

• even with windows applied to the measurement leakage is a very serious problem
• FRFs are generally distorted due to leakage with (significant distortion at the peaks)
• excessive averaging necessary to reduce variance on data
Random Excitation with Hanning Window

Time signal

Frequency Signal

Notice that the coherence is very poor at resonant frequencies
Random Excitation with Overlap Processing

- Used to reduce test time with pure random excitations
- Hanning window tends to weight the first and last quarter of the time block to zero and this data is not effectively used in the normal averaging process
- Effectively uses the portion of the block that has been heavily weighted to zero
- Overlap processing allows for almost twice as many averages with the same data when fifty percent overlap is used
**Pseudo Random Excitation**

An ergodic, stationary signal consisting of only integer multiples of the FFT frequency increment. Signal has constant amplitude with varying phase. Note that the transient part of the signal must decay and steady state response achieved before measurements are taken to assure leakage free FRF.
Pseudo Random Excitation

An ergodic, stationary signal consisting of only integer multiples of the FFT frequency increment. Signal has constant amplitude with varying phase.

ADVANTAGES
- always periodic in the sample interval
- relatively fast
- fewer averages than random
- frequency spectrum is shapeable

DISADVANTAGES
- sensitive to nonlinearities
- same excitation is used for each average
An ergodic, stationary signal consisting of only integer multiples of the FFT frequency increment. Signal has varying amplitude with varying phase. Note that the transient part of the signal must decay and steady state response achieved before measurements are taken to assure leakage free FRF.
Periodic Random Excitation

An ergodic, stationary signal consisting of only integer multiples of the FFT frequency increment. Signal has varying amplitude with varying phase.

ADVANTAGES
- always periodic in the sample interval
- frequency spectrum is shapable
- determines a very good linear approximation of the FRF since leakage is minimized

DISADVANTAGES
- a different signal is generated for each measurement
- longest of all excitation techniques except swept sine
Burst Random Excitation

A random excitation that exists over only a portion of the data block (typically 50% to 70%).

NOTE: Voltage mode amplifier necessary
- creates back emf effect to dampen response at end of burst

Current ~ force
Voltage ~ velocity
Burst Random Excitation

A random excitation that exists over only a portion of the data block (typically 50% to 70%)

ADVANTAGES
- has all the advantages of random excitation
- the function is self-windowing
- no leakage

DISADVANTAGES
- if response does not die out within on sample interval, then leakage is a problem
Burst Random Excitation

Time signal

Frequency Signal

Notice that the coherence is very good even at resonant frequencies
Notice the sharpness of the resonances and measurement quality.
Sine Chirp Excitation

A very fast swept sine signal that starts and stops within one sample interval of the FFT analyzer
Sine Chirp Excitation

A very fast swept sine signal that starts and stops within one sample interval of the FFT analyzer

ADVANTAGES
- has all the same advantages as swept sine
- self windowing function
- good for nonlinear characterization

DISADVANTAGES
- nonlinearities will not be averaged out
**Sine Chirp Excitation**

*Time signal*

*Frequency Signal*

Notice that the coherence is very good.

Notice the sharpness of the resonances and measurement quality.
Digital Stepped Sine Excitation

Sine waves are generated at discrete frequencies which correspond to the digital values of the FFT analyzer for the frequency resolution available. The system is excited with a single sine wave and steady state response measured. Once one spectral line is obtained, the next digital frequency is acquired until all frequencies have been measured.
Digital Stepped Sine Excitation

Sine waves are generated at discrete frequencies which correspond to the digital values of the FFT analyzer for the frequency resolution available. The system is excited with a single sine wave and the steady state response is measured. Once one spectral line is obtained, the next digital frequency is acquired until all frequencies have been measured.

ADVANTAGES

- excellent peak to RMS level
- excellent signal to noise ratio
- good for nonlinear characterization
- leakage free measurements obtained

DISADVANTAGES

- slowest of all test methods
Comparison - Random/Hann, Burst Random, Chirp

RANDOM

BURST RANDOM

SINE CHIRP
When comparing the measurement with random and burst random, notice that the random excitation peaks are lower and appear to be more heavily damped when compared to the burst random. - also notice the coherence improvement at the resonant peaks.
Random with Hanning Window vs Burst Random
Random with Hanning Window vs Burst Random

COH

FRF

RANDOM & HANNING

BURST RANDOM
- Windows will always have an effect on the measured FRF even when the same window is applied to both input and output signals.
- There will always be a distortion at the peak and the appearance of higher damping.
- Windows always, always, always, ... distort data!!!
Linearity Check with Sine Chirp Excitation

ONE FORCE UNIT

FIVE FORCE UNITS

TEN FORCE UNITS
**Shaped Spectrum**

**SHAPEd SPECTRUM EXCITATION**

Uncontrolled broadband excitation techniques are used for most modal testing performed today. However, the relatively flat excitation spectrum causes a wide variation in the response accelerometers. This may be a problem when testing sensitive equipment.

A shaped spectrum, that is controlled, provides an input level that complements the response of the system. This provides a better usage of the ADC since wide variations in level over the frequency range of interest are minimized.
Shaped Spectrum

Random vibration control vs. receiving checkout with shaped spectrum.
Multiple Input Multiple Output
Shaker Testing
Multiple Input Shaker Excitation

Objectives of this lecture:

• Discuss several practical aspects of multiple input multiple output shaker testing

• Discuss some tools commonly used in MIMO testing
Multiple Input Shaker Excitation

- Provide a more even distribution of energy
- Simultaneously excite all modes of interest
- Multiple columns of FRF matrix acquired
- More consistent data is collected
- Same test time as SISO case
Excitation Considerations - MIMO

Multiple referenced FRFs are obtained from MIMO test

Energy is distributed better throughout the structure making better measurements possible
Multiple Input Multiple Output Shaker Testing

\[
\begin{bmatrix}
G_{XF} & H \\
H & G_{FF}
\end{bmatrix}
\]

\[
[H] = \begin{bmatrix}
H_{11} & H_{12} & \cdots & H_{1,N_i} \\
H_{21} & H_{22} & \cdots & H_{2,N_i} \\
\vdots & \vdots & \ddots & \vdots \\
H_{N_o,1} & H_{N_o,2} & \cdots & H_{N_o,N_i}
\end{bmatrix}
\]

Measurements are developed in a similar fashion to the single input single output case but using a matrix formulation.

where

\[
[H] = [G_{XF}][G_{FF}]^{-1}
\]

No - number of outputs

Ni - number of inputs
MIMO Testing – Principal Component Analysis

Check for independent shaker inputs. Perform an SVD on the input shaker matrix commonly called Principal Component Analysis

\[
G_{FF} = U S V^T
\]

The singular values of the SVD should produce large singular values at all frequencies for all shaker excitations. This indicates that the shaker excitation are linearly independent and inversion is possible.
Multiple and Partial Coherence

Two additional coherence functions are needed:

Multiple coherence defines how much of the output signal is linearly related to all of the measured input signals. It is very similar to the ordinary coherence of the single input case.

Partial coherence relates how much of the measured output signal is linearly related to one of the measured input signal with the effects of the other measured input signals removed. All of the partial coherences sum together to form the multiple coherence.
Principal Component Analysis

Check for shaker linear independence
MIMO FRF and Multiple Coherence

Typical MIMO measurements acquired

![Graphs showing MIMO FRF and Multiple Coherence](image-url)
**SISO vs MIMO FRF**

**SISO FRF**

![SISO FRF Graph]

**MIMO FRF**

![MIMO FRF Graph]
Blue Frame - SISO vs MIMO - Reciprocity Checks

RANDOM WITH WINDOW
SINGLE INPUT SINGLE OUTPUT TESTING

MIMO
RANDOM
BURST RANDOM
MULTIPLE INPUT MULTIPLE OUTPUT TESTING
Blue Frame - SISO vs MIMO - Reciprocity Checks

FRFs look reasonably similar

but take a closer look
Notice the variance on the FRF measured and the peak shifting.

**S I S O**

**RANDOM**

**HANNING**
Burst random improves the data but the peaks of the FRFs do not remain the same when single shaker testing is performed.
Blue Frame - SISO vs MIMO - Reciprocity Checks

MIMO random improves the consistency but there are other differences that can be seen at the anti-resonance
Blue Frame - SISO vs MIMO - Reciprocity Checks

MIMO

MIMO burst random improves the data in all respects

BURST RANDOM
Blue Frame - SISO vs MIMO - Reciprocity Checks

The peaks are definitely shifted relative to the SISO and MIMO data

But which is the actual mode ??
Excitation Considerations - MIMO

Large or complicated structures require special attention
Multiple shakers are needed in order to adequately shaker the structure with sufficient energy to be able to make good measurements for FRF estimation.
Excitation Considerations - MIMO

Flimsy Dryer Cabinet requires special attention when measuring frequency response functions for modal testing. Extremely lightweight structures are very difficult to test and obtain quality FRFs.
Excitation Considerations - MIMO

Measurements on the same structure can show tremendously different modal densities depending on the location of the measurement.
Things no one ever told me !!!

Shaker testing is very powerful but there are many issues that must be understood.

Some of these are identified on the next pages
Reciprocity

Even on simple structures, reciprocity can be a problem but not due to the structure.

Here is an example of a stinger flexibility due to rotation effects – the upper portion of the structure has a rotational effect.
Reciprocity - SISO FRF Measurements

Using SISO, several measurements were made at different locations as shown.

While only a few sample measurements are shown, there is an effect of the shaker location on the structure and the rotational stinger effect.
An incorrectly aligned stinger or a poorly fabricated stinger can ruin a test.

Here are two examples of the effect on an FRF measurement due to these problems.
Stinger Length

The length of the stinger can also have an impact on the measured response.

Too short a stinger will have higher lateral stiffness and too long a stinger will have flexibility.
**Stinger Type**

There are many different stinger types

There can be an effect due to these differences
Shaker Excitation

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