Advantages of Multiple-Input Multiple-Output (MIMO) testing using low level excitation systems

M.A. Peres[1], C. Kallmeyer[1], M.C. Witter[2], R. Carneiro[3], F.D. Marques[3], L.P.R. de Oliveira[3]
[1] The Modal Shop, Inc.
3149 E Kemper Rd, Cincinnati, OH 45241, USA
e-mail: **mperes@modalshop.com**

[2] Structural Dynalysis, Ltd.
3300 Diehl Rd, Cincinnati, OH 45211, USA

[3] São Carlos School of Engineering – University of São Paulo
Av. Trabalhador Sancarlense, 400, 13566-590 São Carlos SP, Brazil

Abstract
Multiple-Input Multiple-Output (MIMO) measurement techniques are a well-proven and well-established method for collecting FRF data sets. Despite that, the idea of using multi-references and multiple shakers can be intimidating for an inexperienced modal test engineer. MIMO methods offer some distinct advantages for the measurement and extraction of basic modal parameters especially while testing larger structures. Compared to Single-Input, Single-Output (SISO) techniques, MIMO allows for more uniformly distributed energy across the structure, force levels can be kept lower, modal test data can be taken in one single shot, while nonlinearities are less likely to be excited. Experimental data was taken using single and multiple shakers on an aeronautical structure and results were compared to illustrate the basic principles behind the MIMO technique. Practical aspects of shaker set up, reciprocity checks and the capability of MIMO to resolve closely spaced modes and repeated roots are highlighted throughout the example.

1 Introduction

Experimental modal analysis and vibration testing, in general, have been classic techniques for obtaining the dynamic characteristics of structures, for instance, resonant frequencies, mode shapes and modal damping, providing the basis for further analysis such as: response to a given dynamic load, aeroelastic prediction, model updating/validation and active or passive control design [1,2].

Among the available techniques to excite a structure, the use of shakers is probably the most popular, thanks to its flexibility and capability to reproduce a wide range of excitation signals such as periodic, random or transient. Since the shaker needs to be mechanically attached to the structure under test, it is nearly inevitable that some sort of interaction will occur between them. The causes and effects of this interaction have been an issue for experimentalists since the very beginning of modal analysis [3-5] and is still a relevant research topic, in both open-loop and in closed-loop shaker testing [6-10]. In particular, Oliveira & Varoto [11] presented a study on this matter paying special attention to the force drop-off phenomenon and making a brief review of most of the references cited above. Also, Lang [8] approached the subject from the point of view of the shaker's performance and Peres *et al* [12] presented several practical aspects on setting up the excitation device. The versatility of shakers can also be assessed in less conventional multiple input devices such as 6-DoF exciters [13] often used in environmental and reliability testing.

Even though shaker testing poses some challenges, it outperforms other means of excitation when it comes to non-linear system identification [14] and measuring large complex structures, such as aircraft [15]. While the non-linear system identification is easily performed via SISO or SIMO (Single-Input Multiple-

Output) technique, measuring large complex structures often calls for MIMO testing, which provides a better energy distribution on large structures, allows for multi-directional excitation, a potentially smaller test set-up and aids the proper identification of modal parameters on structures with multiple roots [16].

The aim of the present paper is to present some practical aspects regarding MIMO testing, e.g. reciprocity checks and the capability of MIMO to resolve closely spaced modes and repeated roots. An aeronautical structure has been selected to demonstrate some of those features, namely a helicopter main rotor spider (see Fig.1) built on composite material with some metal inserts. It provides an adequate amount of damping and is presented with a symmetrical shape, leading to closely spaced resonant frequencies (repeated roots). This structure bares a crucial role for the rotor integrity, holding the rotary blades. It also must withstand the cyclic loads and centrifugal forces, rotating at about 300 RPM. The modal analysis presented here could be part of cascade component model validation, providing experimental data for dynamic model correlation.

Figure 1: Helicopter AS350 from Eurocopter - rotor spider location (*based on image taken from* http://www.airbushelicopters.com/)

The remainder of this paper is organized as such: Section 2 deals with the theoretical background on MIMO setup, testing and analysis; a description of the test set-up is given in section 3, while in section 4 we discuss the results focusing on the modal parameters identified with both SIMO and MIMO tests and, last but not least, conclusions are drawn in section 5.

2 Theory and numerical simulation

The fact that MIMO tests use multiple shakers on different degrees of freedom (DoFs) of the structure leads to the following benefits:

i. more evenly distributed energy over the structure
ii. capability of simultaneously excite all modes of interest
iii. FRF matrix with multiple columns
iv. more consistent time invariant data set

As mentioned before, (i) is only perceived on rather big and/or damped structures, while (ii) can be observed even on small structures with 2- or 3-D shapes. Also, structures with localized mode shapes can benefit from MIMO, as different areas can be covered by each shaker; (iii) and (iv) are somewhat related, as having multiple references aids in the quality of the estimated modal parameters, which is also a result of having a more consistent time invariant data set, one that is less susceptible to noise and nonlinear issues than its SIMO counterpart.

In the remainder of this section, a simple numerical exercise is reported in order to show one of the advantages of using multiple inputs in shaker testing. Although not all aspects of a practical implementation can easily be reproduced in simulation, such as poor spatial energy distribution, overtesting and nonlinearity issues, this exercise aims at addressing the drops in the excitation forces. This has been reported several times in the past for single excitation [3-5,7,11] and regarded as an issue for the excitation signal amplitude reduction close to resonant frequencies that may lead to poor signal-to-noise ratios and, hence, poor parameter identification.

Therefore, the main point on the simulation developed below is to show the intrinsic nature of the force drop-off phenomenon when using multiple shakers. The system under study is a 5 DoF mass-spring-damper (see Table 1 for physical properties), excited by two identical shakers, which are modelled as 1DoF mechanical system (shaker armature mass and suspension) (Fig.2) coupled with an RL series electrical circuit, so that the shakers are voltage driven [3,11]. The back electromagnetic voltage generate by the armature motion is also considered, rendering a coupled electromechanical model. Eventually, the shaker armature is excited by the electromagnetic force (F_E) which then drives the structural DoF.

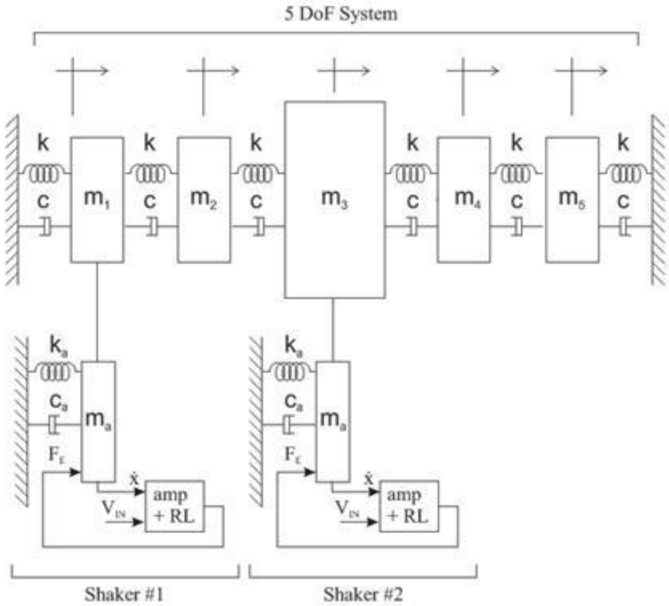


Figure 2 – 5 DoF mechanical system excited by two electromagnetic shakers

parameter	**values**	**unit**
m_1, m_2, m_4, m_5	2.0	kg
m_3	10.0	kg
m_a	0.5	kg
k	4e+5	N/m
k_a	2e+4	N/m
c	5.0	Ns/m
c_a	5.0	Ns/m

Table 1 – lumped parameter system properties

The lumped parameter system is excited on DoFs #1 and #3, as can be seen in Fig.2. The resulting driving point FRFs are shown in Fig. 3, namely, H_{11} and H_{33}. Three excitation configurations are considered: single excitation on DoF #1, again single excitation on DoF #3, and multiple excitations on DoFs #1 and #3. The resulting excitation forces can be seen in Fig. 4. Although DoF #1 shows just some improvement, it can be seen that the force dynamic level on DoF #3 is significantly improved (the force drops are minimized) which in practice would lead to a more assertive data set, less susceptible to noise interference.

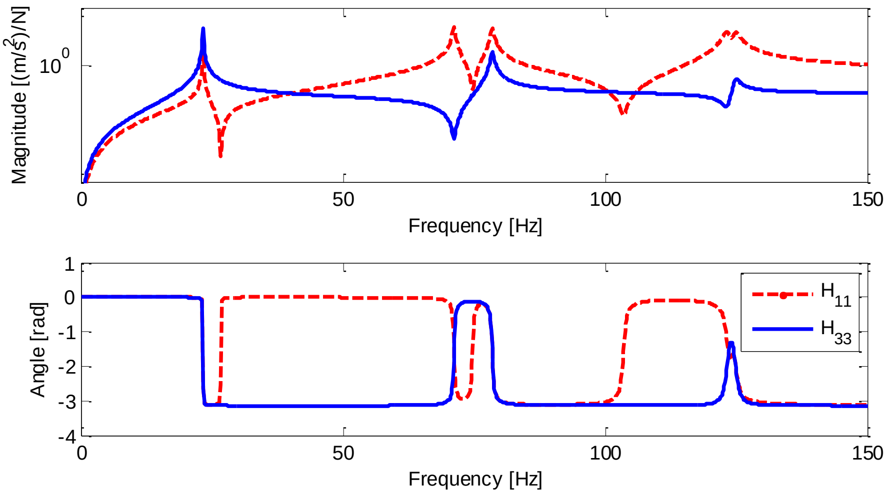


Figure 3 – Lumped parameter system driving point FRFs

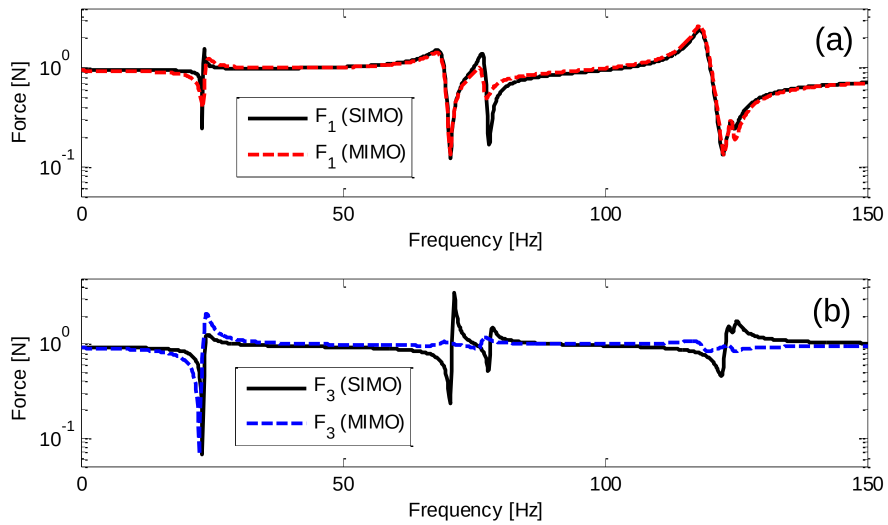


Figure 4 – Excitation forces for SIMO and MIMO cases: (a) Dof #1 and (b) DoF #3

3 Test set-up: helicopter rotor spider

The structure under test is a helicopter rotor spider from an Eurocopter (now Airbus Helicopters) model AS350. It has a triangular shape of approximately 1000 mm in side and 15kg of mass. The helicopter rotor spider is the part responsible for transmitting the rotary motion from the rotor shaft to the rotating blades. The symmetry provided by its triangular shape results in multiple mode shapes lying in, theoretically, the same resonance frequency (multiple or repeated roots). Due to its construction on composite materials, however, one could expect those resonant frequencies to be close, rather than exactly the same.

To perform the experimental modal analysis, the rotor spider was suspended on bungee cords (see Fig.5). A total of 12 unidirectional accelerometers (PCB models 333B and 333B30) were used, while the input forces were measured by impedance heads (PCB model 288D01. The spectral acquisition system is a LMS SCADAS Mobile, with 2 output and 24 input channels, running LMS Test.Lab v.12A.

Two excitation configurations are reported in this paper. The SIMO measurements were conducted with a traditional modal shaker (130 N peak force) powered by a standalone power amplifier running both voltage and current driven modes. During the SIMO measurements, the shaker was mounted on the position indicated as F_1 in Fig.5. The MIMO measurements were carried out using two identical small size SmartShakers (Modal Shop model K2007E01, 31 N peak force) with integrated power amplifier running on voltage mode, mounted on F_1 and F_2. Notice that the position of F_1 in each measurement, SIMO and MIMO, are symmetric, which, in turn, should result in similar FRFs.

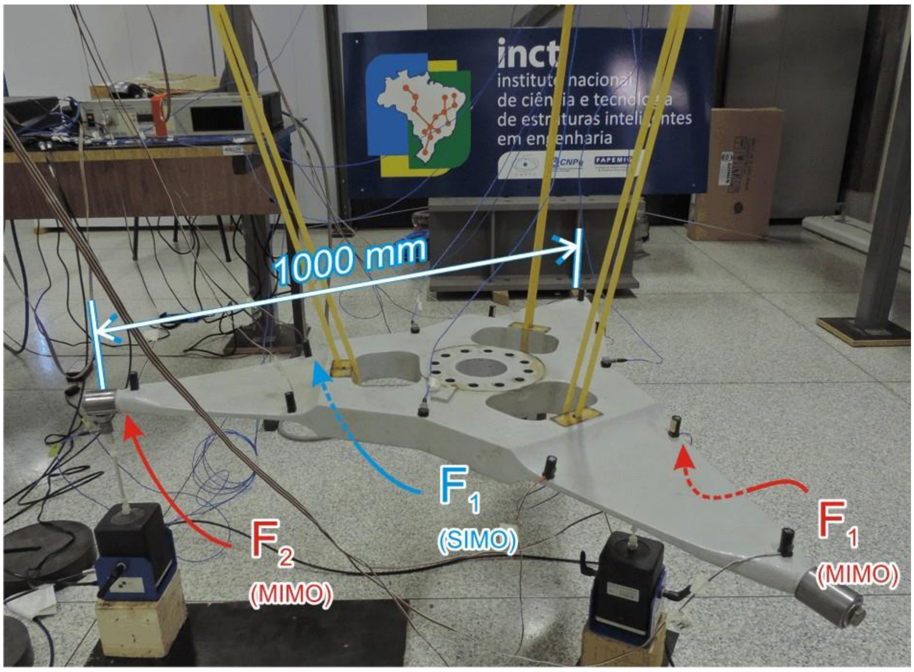


Figure 5 – experimental set-up

The measurement runs, SIMO and MIMO, were conducted with rather distinct levels of excitation, in order to show the trade-offs between one stronger single source of excitation versus multiple less-powerful ones distributed along the test specimen. Figure 6 shows four power spectrum densities, both forces for the MIMO set, SIMO voltage mode and SIMO current mode forces. The overall levels for each case are also indicated in Fig.6, showing 1.72 N_{RMS} and 1.46 N_{RMS} during MIMO testing, while single excitation ranged 7.68 N_{RMS} (current mode) and 13.2 N_{RMS} (voltage mode).

The force drop-offs are clearly seen on both single excitation PSDs, even more pronounced on the current driven test, which also holds the greatest dynamic range. The additional damping provided by the voltage amplifier smoothes out the force spectrum slightly. Besides noise error, one should also be aware of "over testing" (i.e. driving the structure with too much force) when trying to get the most out of a single shaker. In addition to exciting undesirable nonlinearities, sensitive structures or subcomponents can be damaged during testing, which is an important issue, e.g., in aerospace systems [17]. During MIMO testing, however, the force spectra were kept almost flat, as predicted by the simulation, which clearly benefits FRF estimation and reduces measurement uncertainties. The force levels were kept lower for the MIMO test than the SIMO test, to highlight the point that in modal testing distributed smaller forces are typically better than having a single strong one.

Figures 7 and 8 show an overlay of all FRFs obtained in each run, SIMO (voltage mode) and MIMO, respectively. As it can be seen, the structure shows a sort of "band stop" behavior between 100 and 300 Hz. The SIMO FRFs in Fig.7 show two peaks in the range of 50~75Hz and three peaks above 300 Hz. On the other hand, a closer look on Fig.8 shows a more complex dynamic behavior: what seems to be a single peak around 70 Hz for the SIMO measurement, shows two closely spaced peaks in the MIMO run; similarly, between 300 and 400 Hz at least two additional peaks can be found on a quick visual inspection.

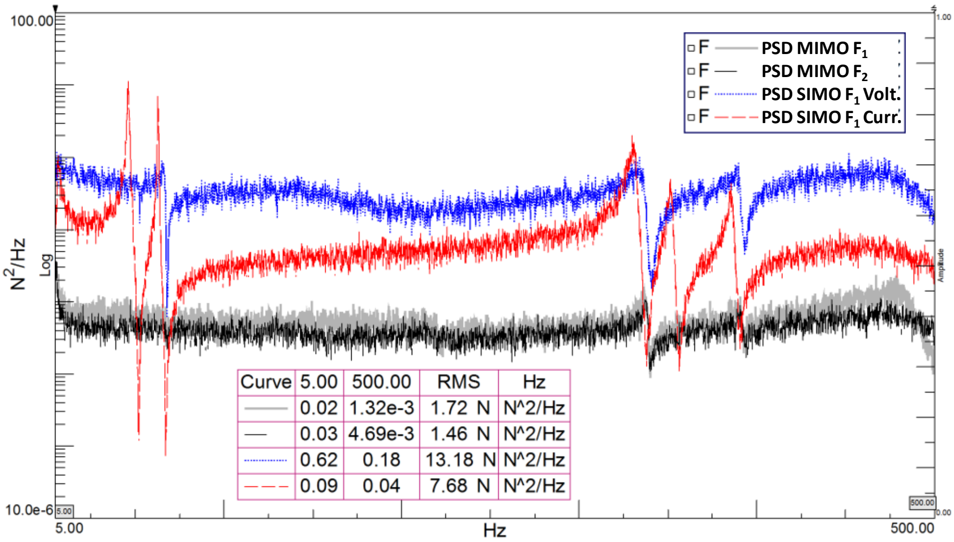


Figure 6 – Force PSDs for SISO and MIMO

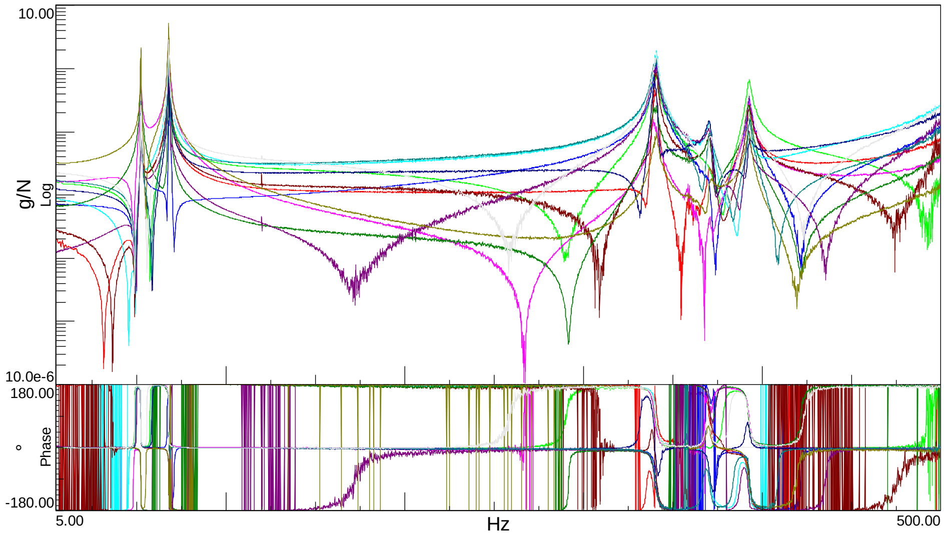


Figure 7 – Collections of FRFs: case SIMO, shaker in current mode

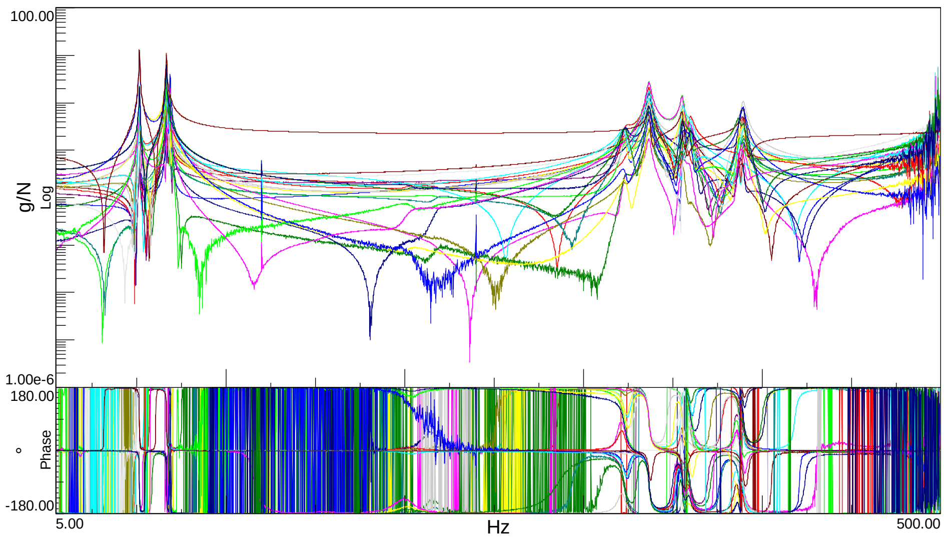


Figure 8 – Collections of FRFs: case MIMO, shakers in voltage mode

One important measure during a MIMO test is checking for reciprocity. Among others, it shows the system still behaves linearly, that the levels on your multiple sources are in agreement, that your excitation signals are sufficiently uncorrelated such that you can indeed have a proper multi-reference analysis, etc. Figure 9 shows three FRFs, the two almost perfectly lined up are the MIMO reciprocal FRFs, while the slightly different one is the SISO. On first glance, there can hardly be any difference between the three FRFs for the first two peaks, although some differences occur for the higher frequency ones. It is known that the shaker assembly can modify the dynamics of the structure, after all they are mechanically attached to structure, however it is important to stress that during MIMO those changes are consistent throughout the data set, while moving a single shaker around will cause a scatter on different FRF runs [16].

Another rather practical aspect of checking for reciprocity, even during pre-testing, is that it provides you instant information on the status of your (maybe many) shakers and correct labeling of your channels. If, for example, one shaker is on standby or a sensor channel is misplaced, some significant differences between those curves will arise.

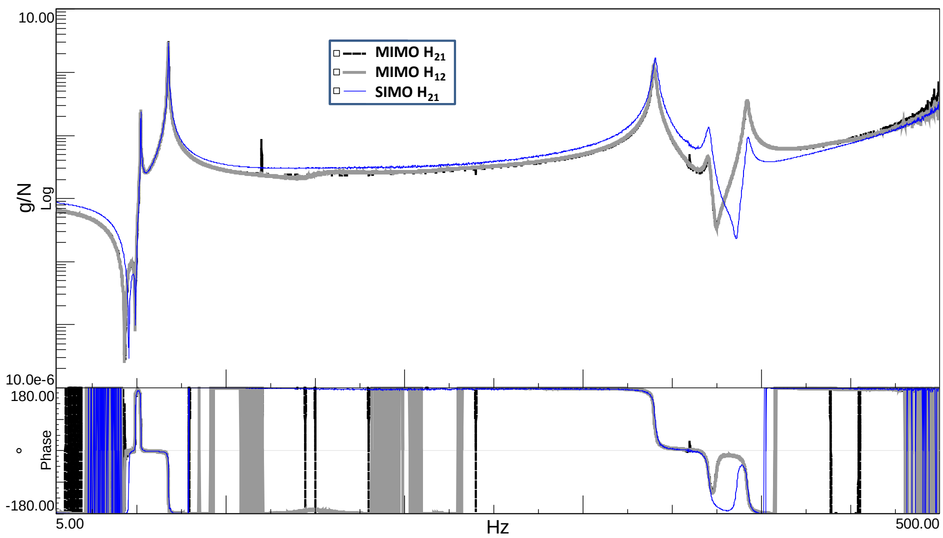


Figure 9 – Reciprocal FRF measurements

4 Modal analysis

In this section, the modal parameters obtained from both SIMO and MIMO runs are obtained and compared. These results are used to illustrate the main advantages of using multiple sources on experimental modal analysis.

The first modal identification method for MIMO systems is the Polyreference Complex Exponential, proposed in the early ´80s [18], which was an extension of the classical Least Square Complex Exponential, originally devised for SIMO applications. Other methods, have soon been proposed [19, 20] including frequency-domain methods that were also available towards the end of the ´80s [21, 22, 23]. More recently, an evolution of the Least Squares Complex Frequency-Domain known as PolyMAX [24], which uses FRFs as primary data, has been introduced. The results presented in this paper have been obtained via the PolyMAX method.

Figure 10 shows the stabilization diagram for the SIMO measurement. As it can be seen, the mode indicator function (MIF) shows a single column of poles for each resonant peak found on the FRF plots (Fig.7), revealing that the single shaker assembly is not capable of sorting out the closely spaced mode shapes that are present.

Figure 11 shows the same plot for the MIMO measurement, presenting a much more populated scatter of stable poles and two MIFs, as there were two references. The modal data obtained from both analyses are summarized in Table 2. For a picture of the mode shapes, please refer to the Appendix.

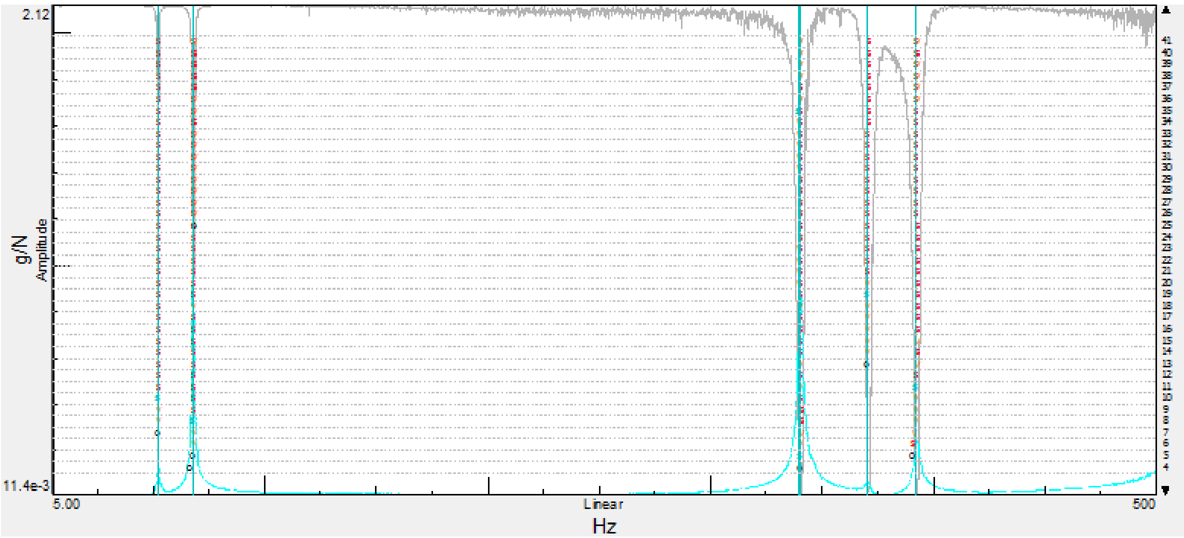


Figure 10 – Stabilization diagram for SIMO measurement

The first column, approximately 52 Hz, represents the first mode shape in which the three legs are bending in phase (see Table 1). This symmetric mode shape presents only one root on both plots as expected. However, in the vicinities of 67 Hz, the MIMO results show two stable poles, instead of one for the SISO. Those two modes are combinations of bending and torsion. There could be even more poles around there, would there be more excitation channels available.

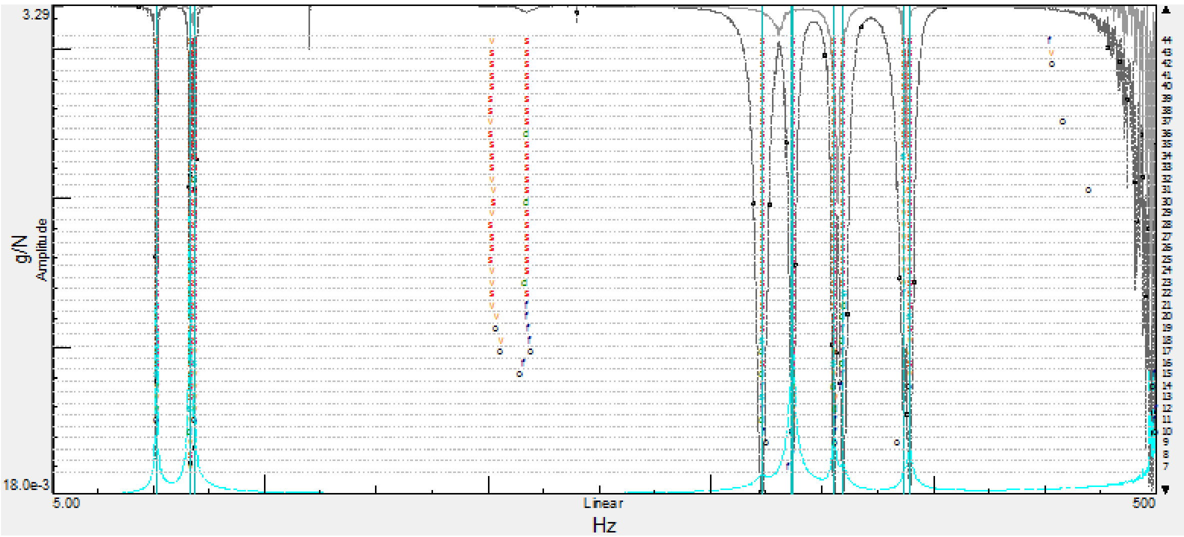


Figure 11– Stabilization diagram for MIMO measurement

In between 100 ~ 300 Hz, some poles appear on the MIMO stabilization diagram, which are judged as numerical artifacts present, maybe, due to the low level of excitation selected for this run.

Not only four depths are present in the MIMO MIFs between 300 ~ 400 Hz (while only three on the SIMO MIF), but also some of them show closely spaced poles, adding up to 11 modes identified in a single MIMO run, versus only 6 on the SIMO. By visual inspections of the mode shapes, a total of 12 modes are depicted in the Appendix, with modal data described in Table 2. Some modes, numbered successively, are symmetric pairs or trios, such as modes 6 and 7 or modes 10, 11 and 12. When the same mode shape is found on both analyses, their resonance frequencies and modal damping ratios are attributed to the same mode number, which is the case of modes 1, 3, 6 and 9.

	MIMO		SIMO	
Mode #	**Freq. [Hz]**	**damping**	**Freq. [Hz]**	**damping**
1	51.6	0.23%	52.4	0.16%
2	66.6	0.27%		
3	68.8	0.19%	67.9	0.23%
4	323.0	0.59%		
5	323.2	0.60%		
6	336.7	0.30%	339.9	0.22%
7	337.3	0.27%	340.5	0.34%
8	355.5	0.23%		
9	359.6	0.43%	370.5	0.31%
10	387.0	0.29%		
11	389.2	0.25%		
12			392.4	0.22%

Table 2 – Modal data from SIMO and MIMO measurements

The complete identification of such a small and rather flat structure is not even achieved with the two shaker set-up. As the single excitation force was placed on a symmetric position, with regard to F_1 in the MIMO run, complementary mode shapes have been identified. For a full modal identification, further measurements would still have to be done. However, for the sake of this study, the results presented here are sufficient to demonstrate the value of multiple shaker excitation, even with much lower excitation levels than the single input.

5 Conclusions

Some advantages of MIMO over SIMO testing have been demonstrated, numerically and experimentally. The lumped parameter model simulation reveals a key advantage of MIMO, as the force spectra are smoothened in the presence of multiple shakers.

To demonstrate it experimentally, a helicopter rotor spider is subjected to single and multiple shaker excitation, while supported in free-free boundary condition. Firstly, the experiments corroborate the smoothened force spectra, predicted by the simulation. The reciprocal FRFs taken during the MIMO tests show a good agreement, which is also a sign of a good quality data set.

The experimental data also shows that low level distributed MIMO measurements provided much better, complete multi-reference FRFs, allowing the identification of closely spaced modes, not evident in the SIMO measurements. And, specifically applying a larger force at a single input location degraded the quality of the data set due to force function drop-offs and nonlinearities.

Finally, parameter estimation has been performed on both data sets, revealing quite some advantages of the MIMO over the SIMO, e.g. the number of stable poles detected via the MIMO method.

Acknowledgements

The authors acknowledge the financial support of the Brazilian National Council for Scientific and Technological Development (CNPq), Grants 307369/2013-7 and 574001/2008-5 - National Institute of Science and Technology on Smart Structures in Engineering.

References

[1] M. Migdalovici, T. Sireteanu, E.M. Videa, *Control of vibration of transmission lines*, International Journal of Acoustics and Vibrations 15 (2) 65-71 (2010).

[2] S.J. Elliott, *Active control in vehicles and in the inner ear: A review*, International Journal of Acoustics and Vibrations 14 (4) 212-219 (2009).

[3] K. G. McConnell, P.S. Varoto, *Vibration Testing: Theory and Practice*, John Wiley & Sons, NY, EUA, (2008).

[4] K. Unholtz, *Vibration testing machines - Shok and Vibration Handbook*, McGraw-Hill Book Co., New York, (1961) ed.1, Vol.2, pp. 25.1 - 25.74.

[5] G. R. Tomlinson, *Force Distortion in Resonance Testing of Structures with Electrodynamic Vibration Exciters*, Journal of Sound and Vibration, 63(3) 337-350 (1979).

[6] T. Olbrechts, P. Sas, D. Vandepitte, *FRF measurements errors caused by the use of inertia mass shakers*, Proceedings of the 15 International Modal Analysis Conference, IMAC-1, 188-194 (1997).

[7] G. F. Lang, *Electrodynamic Shaker Fundamentals*, S/V Sound and Vibration 31(4) 14-23 (1997).

[8] G. F. Lang, *Understanding the Physics of Electrodynamic Shaker Performance*, S/V Sound and Vibration 35(10) 24-33 (2001).

[9] P. Van Baren, G.F. Lang, *Examining the dynamic range of your vibration controller*, S/V Sound and Vibration 41(8) 12-21 (2007).

[10] G.F. Lang, G Burroughs, G Carmichael, *Random Testing to New Limits*, S/V Sound and Vibration, 43(2) 5-9 (2009).

[11] P.S. Varoto, L.P.R. de Oliveira, *On the Force Drop-off Phenomenon in Shaker Testing in Experimental Modal Analysis*, Journal of Shock and Vibration 9(4-5 SPEC.) 165-175 (2002).

[12] M.A. Peres, R.W. Bono, D.L. Brown, *Practical Aspects of Shaker Measurements for Modal Testing*, Proceedings of ISMA2010 – Leuven, Belgium 2010, p.1-12.

[13] F. De Coninck, D. Vaes, J. Swevers, W. Desmet, P. Sas, *Vehicle suspension and structure borne road noise testing on a 6-DOF high frequency shaker table: first outlines*, Proceedings of the 30th Fisita 2004 World Automotive Congress, Barcelona - Spain: 5pp.

[14] J. Schoukens, J. Swevers, R. Pintelon, H. Van der Auweraer, *Excitation design for FRF measurements in the presence of nonlinear distortions*, Proceedings of the International Conference on Noise and Vibration Engineering ISMA2002, Leuven – Belgium, 2002, p. 951-958.

[15] B. Peeters, *et al*. *Modern solutions for Ground Vibration Testing of large aircraft* Proceedings of the 26th International Modal Analysis Conference - IMAC 26, Orlando – USA, 2008, 14pp.

[16] P. Avitabile, *Is there any real advantage to MIMO testing?* SEM Experimental Techniques - August 2002, p.1-2.

[17] Y.K. Kim, K.W. Kim, H.B. Kim, *Review experiments on NASA RP-1403 'Force limited vibration testing monograph'* Proceedings of the 5th International Symposium on Environmental Testing for Space Programmes, Noordwijk – The Netherlands, 2004, 217-221.

[18] H. Vold, J. Kundrat, T. Rocklin, R. Russell, *A multi input modal parameter estimation algorithm for a mini computer*, Society of Automotive Engineers, Paper 820194 (1982)

[19] J. Leuridan, H. Vold, *Time domain linear model estimation technique for global modal parameter identification*, Proceedings of the 2nd International Modal Analysis Conference & Exhibit, Orlando, FL, USA, Volume 1, 1984, Pages 443-451.

[20] L.M. Zhang, Y.X. Yao, M.F. Lu, *An Improved Time Domain Polyreference Method for Modal Identification*, Mechanical System and Signal Processing, 1(4)1987.

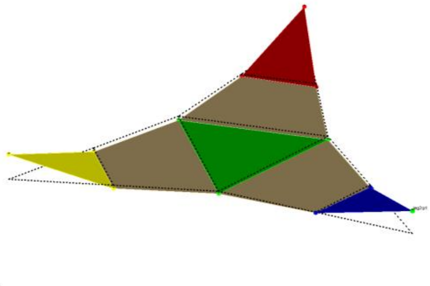
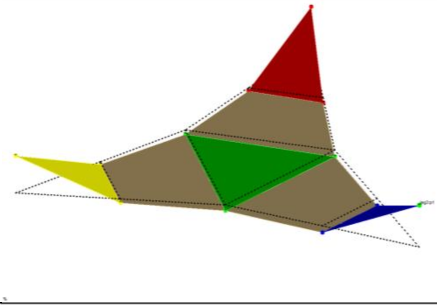
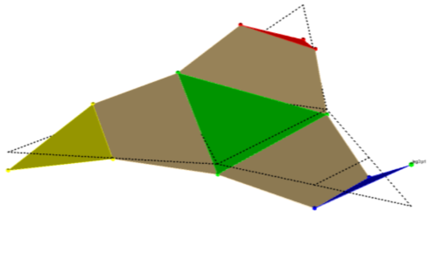
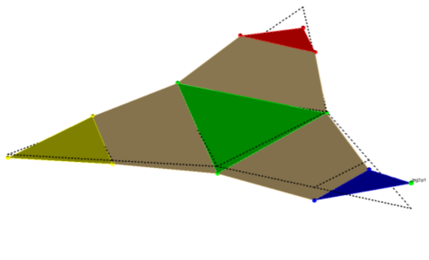
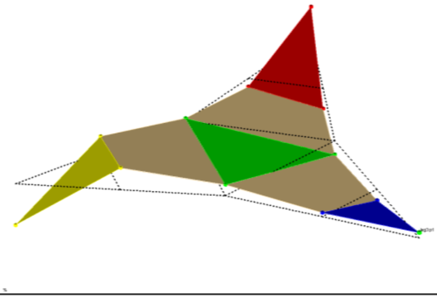
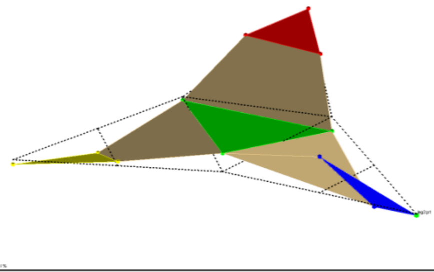
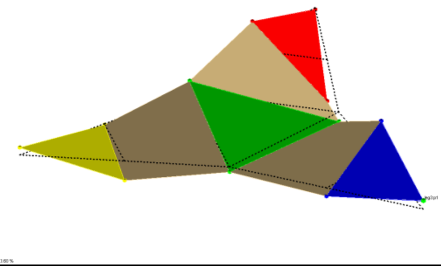
[21] L.M. Zhang, H. Kanda, D.L. Brown, R.J. Allemang, *A frequency domain polyreference method for modal analysis*, ASME Paper 85-Det-106, 1985, p.1-6.

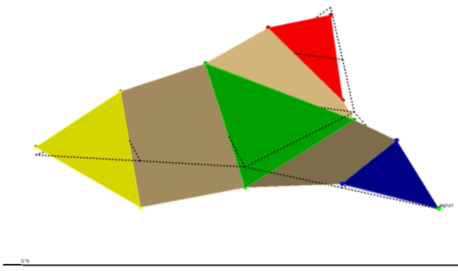
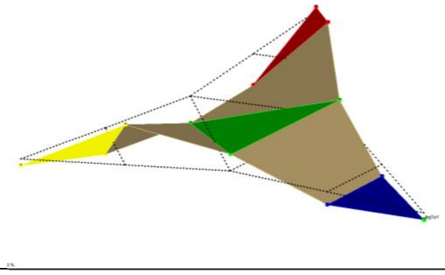
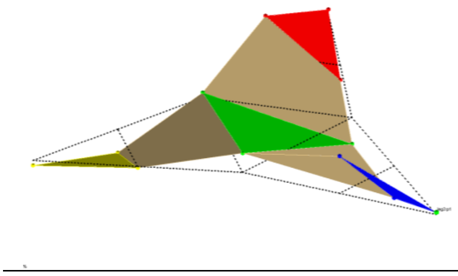
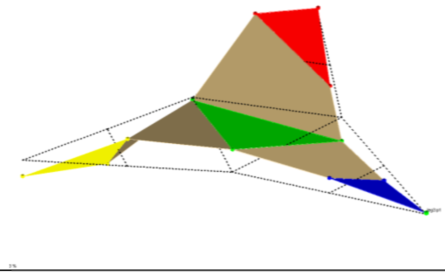
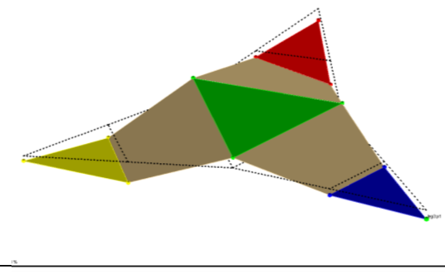
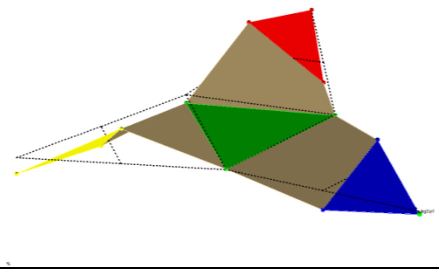
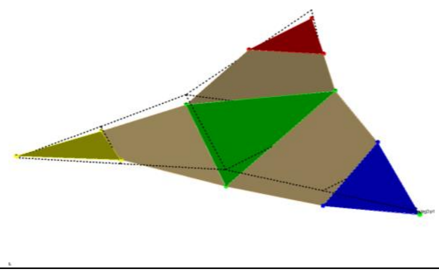
[22] F. Lembrets, J. Leuridan, L.M. Zhang, H. Kanda, *Multiple Input Modal Analysis of Frequency Response Functions based on Direct Parameter Identification*, Proceedings of the International Modal Analysis Conference - IMAC 4, 1986, p.589-598.

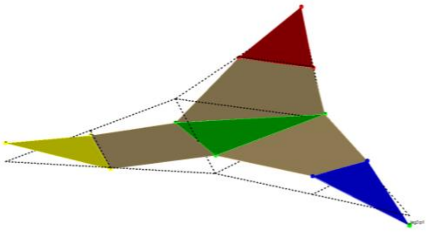
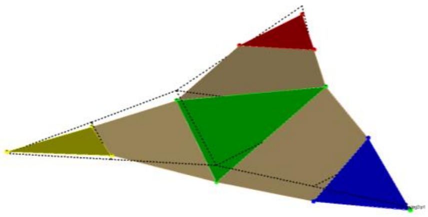
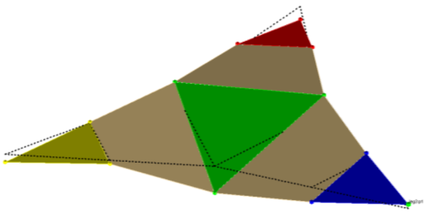
[23] C.Y. Shih, Y.G. Tsuei, R.J. Allemang, D.L. Brown, *A Frequency Domain Global Parameter Estimation Method for Multiple Reference Frequency Response Measurements*, Mechanical Systems and Signal Processing, 2(4) 1988, p. 349-365.

[24] B. Peeters, G. Lowet, H. Van der Auweraer, J. Leuridan, *A new procedure for modal parameter estimation*. Sound and Vibration, 38(1), 2004, 24-29.

Appendix: mode shapes

Mode shape SISO	Mode Shape MIMO	Modal Data
		Mode #1 SIMO f = 52.4 Hz ζ = 0.16% MIMO f = 51.6 Hz ζ = 0.23%
		Mode #2 MIMO f = 66.6 Hz ζ = 0.27%
		Mode #3 SIMO f = 67.9 Hz ζ = 0.23% MIMO f = 68.8 Hz ζ = 0.19%
		Mode #4 MIMO f = 323.0 Hz ζ = 0.59%
		Mode #5 MIMO f = 323.2 Hz ζ = 0.60%

Mode shape SISO (cont.)	Mode Shape MIMO (cont.)	Modal Data (cont.)
		Mode #6 SIMO f = 339.9 Hz ζ = 0.22% MIMO f = 336.7 Hz ζ = 0.30%
		Mode #7 SIMO f = 340.5 Hz ζ = 0.34% MIMO f = 337.3 Hz ζ = 0.27%
		Mode #8 MIMO f = 355.5 Hz ζ = 0.23%
		Mode #9 SIMO f = 370.5 Hz ζ = 0.31% MIMO f = 359.6 Hz ζ = 0.43%

Mode shape SISO (cont.)	Mode Shape MIMO (cont.)	Modal Data (cont.)
		Mode #10 MIMO f = 387.0 Hz ζ = 0.29%
		Mode #11 MIMO f = 389.2 Hz ζ = 0.25%
		Mode #12 SIMO f = 392.4 Hz ζ = 0.22%